Menoufia University الفرقة الثالثة اتصالات ع.م
Faculty of Electronic Engineering
Dept. Electronics and Electrical Commun. Eng.
Course: Elective Course (1) (ECE 315)
C1: Random variables and Random Processes
Total Exam Time: 3 Hours (10:00 AM-01:00 PM)

جامعة المنوفية

Final Exam
Academic Year: 2019/ 2020
Date: 22 / 01 / 2020
Third Year
No. of Exam Questions: 3
Total Exam Marks: 70 Marks

Answer all the following questions:

Question 1: [25 Marks]

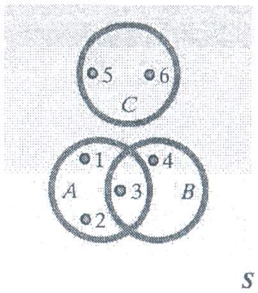
a) Explain briefly the meaning of the following terms: [5 Marks]

a. Experiment.
b. Event.
c. Simple event.
d. Mutually exclusive events.
e. Sample space.

b) Two fair dice are tossed, and the up face on each die is recorded. [10 Marks]

a. Calculate the number of sample points.
b. Give the probabilities of different sample points.
c. Determine the probabilities of the following events:
   i. A: {A 3 appears on each of the two dice}.
   ii. B: {The sum of the numbers is even}.
   iii. C: {The sum of the numbers is equal to 7}.
   iv. D: {A 5 appears on at least one of the dice}.
   v. E: {The sum of the numbers is 10 or more}.

c) A sample space contains six sample points and events A, B, and C, as shown in this Venn diagram. The probabilities of the sample points are $P(1) = 0.2$, $P(2) = 0.05$, $P(3) = 0.3$, $P(4) = P(5) = 0.1$, and $P(6) = 0.25$. [10 Marks]



a. Which pairs of events, if any, are mutually exclusive? Why?
b. Which pairs of events, if any, are independent? Why?
c. Find $P(A \cup B)$ by adding the probabilities of the sample points and then by using the additive rule. Verify that the answers agree. Repeat for $P(A \cup C)$.

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Question 2: [25 Marks]

a) Given that the pdf of a Gaussian random variable X is $p_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$, where μ is the mean and σ^2 is the variance, derive an expression for the characteristic function of X. [5 Marks]

b) Suppose that X is a zero-mean unit-variance Gaussian random variable. Let Y be a random variable given by $Y = aX^3 + b, \quad a > 0$. Determine the probability density function (pdf) of Y. [10 Marks]

c) Given that the characteristic function of the central chi-square random variable with n degrees of freedom can be expressed as: [10 Marks]

$$\psi(jv) = \frac{1}{(1 - j2v\sigma^2)^{n/2}}$$

Determine the corresponding first and second moments.

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Question 3: [20 Marks]

a) Given that the probability density function (pdf) of a Cauchy distributed random variable X is: [10 Marks]

$$p_X(x) = \frac{a/\pi}{x^2 + a^2}, \quad -\infty < x < \infty.$$

Determine the mean and the variance of X.

b) Consider the following sinusoidal process: [10 Marks]

$$X(t) = A\cos(2\pi f_c t),$$

where the frequency f_c is constant and the amplitude A is uniformly distributed as:

$$f_A(a) = \begin{cases} 1, & 0 \le a \le 1 \\ 0, & otherwise \end{cases}$$

Determine whether or not this process is strictly stationary.

WITH MY BEST WISHES **DR. AHMED MOHAMED BENAYA**