A THEORETICAL ANALYSIS FOR THE CHARACTERISTICS OF HUMAN ARTICULAR JOINTS

التحليل النظرى لخواص الوصلات المفصلية الآدمية

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الخلاصة ـ هذا البحث يقدم دراسة نظرية للوصلات المفصلية الآدمية وذلك بعمل نموذج مبسط لها عبارة عن مستويين متوازيين وبينهما سائل سينوفلى . وقد تم دراسة تأثير العوامل التالية على هذه الوصلات : التغير فى الضغط ، سعة التحميل ، زمن الاقتراب ، تركيز سائل التزليق ، أقل سمك لطبقة السائل وكذلك تأثير مسامية الغضاريف على الترلىق . وقد اعتمد هذا التحليل الرياضى على استمرارية الضغط وسرعات الاسطح البينية لسمك طبقة السائل السينوفلى وكذلك الشرائح الثلاثية للغضاريف مختلفة المسامية . وقد أوضحت الدراسة نتائج هامة وقيمة ومطابقة للملاحظات التجريبية السابقة .

ABSTRACT - A simple model of two parallel plates with reference to human articular joints was suggested and studied in the presence of synovial fluid. Variation of pressure, load capacity, time of approach, concentration of lubricant, minimum film thickness and the effect of cartilage porosity upon lubrication, have been investigated. The mathematical analysis was based on the continuity of pressure and velocities at the interface of the synovial fluid film and the three cartilage layers possessing different porosities. The study brings out many valuable and important results which are in good agreement with earlier experimental observations.

NOTATION

a	characteristic length of the bearing
A	dimensionless shape and size index of micro-particle
B	characteristic width of the bearing
h	film thickness
h_o	initial film thickness
$\bar{h}$	dimensionless film thickness (h/h_o)
$H1, H2, H3$	thickness of the upper, middle and lower layers of cartilage
$k1, k2, k3$	porosity of the upper, middle and lower layers of cartilage
k	non-dimensional porosity parameter (k/h_o^2)
l	$(Y/4\mu)^{1/2}$
L	h_o/l
N	coupling number $[X/(2\mu+X)]^{1/2}$
p	pressure in fluid film region
$\bar{p}$	non-dimensional pressure in fluid film region $(8ph_o^3/\mu Va^2)$
$p1, p2, p3$	pressure of upper, middle and lower layers of cartilage
R	dimensionless concentration index of micromolecules
t	time taken to reduce the film thickness from h_o to h
T	dimensionless time of approach $(2Wth_o^2/ua^3B)$
u, v	velocity components in fluid film region
$\bar{u}, \bar{v}$	velocity components in porous matrix of cartilage
V	velocity of approach (dh/dt)

W	load capacity
$\bar{W}$	dimensionless load capacity $(12Wh^3/\mu V a^3 B)$
x	x-coordinate
y	y-coordinate
μ	Newtonian viscosity coefficient
Y, X	viscosity coefficients for a micropolar fluid
ω	micro-rotation velocity

1. INTRODUCTION

The natural synovial joint is a remarkable bearing. It is expected to function in the human body for a period of about sixty years while transmitting quite large dynamic loads and yet accommodating a wide range of movements with extremely low coefficient of friction (as low as 0.005) and low wear rate.

In recent years tribologists have worked with rheumatologists, orthopaedic surgeons and biochemists in an effort to understand the remarkable characteristics of healthy synovial joints. For example, several types of lubrication mechanisms, for the human articulating synovial joints, have been described: hydrodynamic [1], boundary [2], weeping [3], boosted [4] and mixed lubrication [5]. However, it now appears that attempts to attribute a single mode of lubrication to synovial joints may have been misguided; for they appear to enjoy all the common modes of lubrication known to engineers [6] and perhaps some novel ones, in their varied and arduous existence.

In synovial human joints, under loaded conditions, the load can be supported by both the cartilage matrix and the synovial fluid which exists between cartilage surfaces within the joint cavity. It is known that synovial fluid plays an important role in reducing both friction and wear of joints. Previous work concerning the study of synovial fluid variation of viscosity [7], or describing it as non-Newtonian fluid [8] have failed to explain the reasons for the variation in size and shape of the hyaluronic acid molecules which form the main constituent of synovial fluid and the increase of synovial viscosity near the cartilage surface while the joint is loaded. Therefore, in the present study, the continuum mechanics approach was utilised to describe the microscopic events taking place in synovial joint.

In synovial joint, the bone end is covered with a layer of relatively soft and porous articular cartilage. The thickness of the cartilage varies from joint to joint, from one individual to another and with time. The layer of articular cartilage is elastic and the effective elastic modulus is time dependent. McCutchen [9] has quoted Young's modulus values in the range 10^6 - 10^8 dynes/cm². Another important feature of cartilage is its porosity, which enables it to imbibe or exude synovial fluid. Cartilage matrix is known to consist of three layers of different porosities: a superfacial zone of thickness H1, a middle zone of thickness H2 and a deep zone in contact with the underlaying bone and of thickness H3. Fig. 1. demonstrates the structure of articulating cartilage and the three layers.

The synovial fluid is essentially a dialysate of blood plasma with the addition of non-sulphated mucopolysaccharide hyaluronic acid which has a molecular weight of about 10^6 . The viscous properties of the fluid seem to be dictated by the hyaluronic acid molecules (molecular length of 5000-10000 °A). The fluid is non-Newtonian with long chain polymers present.

The motivation for the present analysis is to study a simple model describing the normal approach of a rigid solid and a porous one consisting of three layers of different porosities separated by micropolar fluid with a particulate suspension whose concentration increases as the gap between the approaching surfaces decreases. The suspending medium (viscous fluid) enters into the porous matrix of the solid as the surfaces approach each others. The model has been analysed by taking the continuity of pressure and velocities at the interface of the fluid film and the porous layers.

2. MATHEMATICAL FORMULATION FOR THE PROPOSED MODEL

At the fluid film region, Eringen [10] has derived the following equations for the pressure variation of a micropolar fluid:-

$$\frac{dp}{dx} = \frac{(2\mu + X)}{2} \frac{\partial^2 u}{\partial y^2} + X \frac{\partial \omega}{\partial y} \quad \text{...(1)}$$

$$\frac{dp}{dy} = Y \frac{\partial^2 \omega}{\partial y^2} - 2 X \omega - X \frac{\partial u}{\partial y} = 0 \quad \text{...(2)}$$

The boundary conditions beeing:

$$\begin{aligned} &\text{at } y = 0 \quad & v = \bar{v},\ u = \bar{u},\ \omega = 0 \\ &\text{at } y = h \quad & v = V,\ u = 0,\ \omega = 0 \\ &\text{at } x = \pm a/2 \quad & p = 0 \end{aligned} \quad \text{...(3)}$$

Within the matrix of cartilage, the viscous fluid in the porous matrix is described by Darcy's Law:-

$$\bar{u} = \frac{-k_i}{\mu} \frac{\partial p_i}{\partial x} \quad (i = 1, 2, 3 \text{ for the three layers of cartilage}) \quad \text{...(4)}$$

and

$$\bar{v} = \frac{-k_i}{\mu} \frac{\partial p_i}{\partial y} \quad \text{..(5)}$$

The boundary conditions for the porous region beeing:-

$$\text{at } x = \pm a/2 \quad p_i = 0 \quad (i = 1, 2, \text{and } 3) \quad \text{...(6)}$$

$$\text{at } y = -(H1 + H2 + H3) \quad \frac{\partial p_3}{\partial y} = 0 \quad \text{..(7a)}$$

$$\frac{k_1}{\mu}\left(\frac{\partial p_1}{\partial y}\right)_{y=-H1} \quad \frac{k_2}{\mu}\left(\frac{\partial p_2}{\partial y}\right)_{y=-H1} \quad \text{..(7b)}$$

$$\frac{k_2}{\mu}\left(\frac{\partial p_2}{\partial y}\right)_{y=-(H1+H2)} = \frac{k_3}{\mu}\left(\frac{\partial p_3}{\partial y}\right)_{y=-(H1+H2)} \quad \text{..(7c)}$$

The conditions expressed by equations (7b) and (7c) result from the continuity of pressure at the inner layers of the porous cartilage matrix.

3. SOLUTION OF THE MATHEMATICAL MODEL

To solve the mathematical model, the following assumptions are made:-

A) The porous cartilage layers are homogeneous and isotropic.

B) The synovial fluid lubricant in the porous region is an incompressible Newtonian fluid.

For the fluid film region, by differentiating equation (2) with respect to y and solving the resulting equation with equation (1), we obtain:

$$\frac{\partial^3 \omega}{\partial y^3} - m^2 \frac{\partial \omega}{\partial y} = \frac{m^2}{2\mu}\left(\frac{\partial p}{\partial x}\right) \quad \text{...(8)}$$

where

$$m = \sqrt{\left[\frac{4\mu X}{Y(2\mu + X)}\right]} = \frac{N}{l}$$

$$N = \sqrt{\left(\frac{X}{2\mu + X}\right)}, \quad \text{and} \quad l = \sqrt{\left(\frac{Y}{4\mu}\right)}$$

Solving equation (8), we obtain :

$$v = c1 \cosh my + c2 \sinh my - \frac{y}{2}\frac{dp}{dx} - \frac{A}{m^2}$$

but $\omega = 0$ at $y = 0$ and $y = h$

therefore:

$$\omega = \frac{A}{m^2}\left[\sinh m(h-y) + \sinh my - \sinh mh\right]/\sinh mh +$$

$$+ \frac{1}{2\mu}\frac{dp}{dx}\left[\frac{h \sinh my}{\sinh mh} - y\right] \quad ...(9)$$

Substituting the value of ω in equation (2) we obtain equation (10) as follows:

$$\frac{\partial u}{\partial y} = \frac{y}{\mu}\cdot\frac{dp}{dx} - \frac{Dhm}{2\mu}\frac{dp}{dx}\frac{\sinh my}{\sinh mh} + \frac{A[2\sinh mh - Dm \sinh m(h-y) - Dm \sinh my]}{m^2 \sinh mh} \quad ...(10)$$

where

$$D = \frac{2}{m} - \frac{Y\,m}{y}$$

Integrating equation (10) with respect to y and using the conditions of equation (3) we obtain :

$$A = F(h)\,.\frac{dp}{dx} \quad ...(11)$$

where

$$F(h) = \frac{h\,m^2}{4\mu}\cdot\frac{D(1 - \cosh mh) + [h - (2k1/h)]\sinh mh}{D(\cosh mh - 1) - h \sinh mh}$$

and

$$u = \frac{y^2}{2\mu}\cdot\frac{dp}{dx} - \frac{k1}{\mu}\cdot\frac{dp}{dx} + \frac{Dh}{2\mu}\cdot\frac{(1 - \cosh my)}{\sinh mh}\cdot\frac{dp}{dx}$$

$$+ F(h)\frac{dp}{dx}\left[\frac{2y \sinh mh + D\cosh m(h-y) - D\cosh my - D(\cosh mh - 1)}{m^2 \sinh mh}\right] \quad ...(12)$$

Applying continuity equation in the porous region to equations (4) and (5) :

The continuity equation $\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0$

we obtain:- $\Delta^2\, pi = 0$

i.e. the pressures in the porous region satisfy the Laplace equation. Thus, p1 the pressure in the upper porous layer of thickness H1 satisfies the equation:

$$\frac{\partial^2 p1}{\partial x^2} + \frac{\partial^2 p1}{\partial y^2} = 0 \quad ...(13)$$

Integrating equation (13) with respect to y over the layer thicknesses H1, H2 and H3 and using the conditions in equation (7b), we obtain:-

For the upper layer of thickness H1:

$$\left(\frac{\partial p1}{\partial y}\right)_{y=0} = -\int_{-H1}^{0} \frac{\partial^2 p1}{\partial x^2} dy + \frac{k2}{k1} \left(\frac{\partial p2}{\partial y}\right)_{y=-H1} \quad ...(14)$$

For the middle layer of thickness H2:

$$\left(\frac{\partial p2}{\partial y}\right)_{y=-H1} = -\int_{-(H1+H2)}^{-H1} \frac{\partial^2 p2}{\partial x^2} dy + \frac{k3}{k2} \left(\frac{\partial p3}{\partial y}\right)_{y=-(H1+H2)} \quad ...(15)$$

For the deep layer of thickness H3:

$$\left(\frac{\partial p3}{\partial y}\right)_{y=-(H1+H2)} = -\int_{-(H1+H2+H3)}^{-(H1+H2)} \frac{\partial^2 p3}{\partial x^2} dy \quad ...(16)$$

From equations (14), (15) and (16) we obtain:

$$\left(\frac{\partial p1}{\partial y}\right)_{y=0} = -\int_{-H1}^{0} \frac{\partial^2 p1}{\partial x^2} dy - \frac{k2}{k1} \int_{-(H1+H2)}^{-H1} \frac{\partial^2 p2}{\partial x^2} dy - \frac{k3}{k1} \int_{-(H1+H2+H3)}^{-(H1+H2)} \frac{\partial^2 p3}{\partial x^2} dy \quad ...(17)$$

If the layer thicknesses H1, H2 and H3 are assumed to be small, equation (17) could be reduced to:

$$\left(\frac{\partial p1}{\partial y}\right)_{y=0} = -\left(H1 + \frac{k2}{k1} H2 + \frac{k3}{k1} H3\right) \cdot \frac{\partial^2 p}{\partial x^2} \quad ...(18)$$

It is shown by Srinivasan [11] that for the case $H1 \to 0$, $H2 \to 0$ and $H3 \to 0$, equation (18) is valid exactly. Therefore, for small values of H1, H2 and H3, i.e., for the case when the porous layers are thin, this approximation is not likely to cause significant error.

A) The Pressure Equation

From the equation of continuity and equation (18) we get:

$$\frac{\partial}{\partial x}\int_{0}^{h} u \, dy = H \frac{d^2 p}{d x^2} + V \quad ...(19)$$

where

$$H = [\, H1 + (k2/k1)\, H2 + (k3/k1)\, H3 \,]$$

Integrating with respect to x and using the boundary conditions (3), the pressure is given by:-

$$P = \frac{1.5\, V a^2 (1 - 4 x^2 / a^2)}{h\,[\, h^3 + (6D/m) + 6\, k1 + (12\, \mu\, H/h) - 3\, Dh \coth (mh/2) \,]} \quad ...(20)$$

The non-dimensional pressure distribution is given by:-

$$\bar{P} = \frac{1 - 4\,\bar{x}^2}{F(\bar{h}, \bar{H}, L, \bar{k}1, N)} \quad ...(21)$$

where

$x = \bar{x}/a$, $\quad \bar{k}1 = k1/ h_0^3$ $\quad \bar{H} = G\, H1/ h_0$

$G = 1 + (k2\, H2/ k1\, H1) + (k3\, H3/ k1\, H1)$

$$F(\bar{h}, \bar{H}, L, \bar{k1}, N) = \bar{h}^3 \left[\frac{1}{12} + \frac{1}{\bar{h}^2 L^2} + \frac{\bar{k1}}{\bar{h}^3} \left(\frac{1}{2} + \frac{\bar{H}}{\bar{h}} \right) - \frac{N}{2L\bar{h}} \coth \left(\frac{NL\bar{h}}{2} \right) \right]$$

B) The Load-carrying Capacity

The load-carrying capacity of the joint is given by:

$$W = B \int_{-a/2}^{a/2} P \, dx$$

$$W = \frac{V a^3 B}{h \left[h^2 + (6D/m) + 6\, k1 + (12\, \mu H/h) - 3Dh \coth (mh/2) \right]} \quad \ldots(22)$$

The non-dimensional load-carrying capacity is:

$$\bar{W} = \frac{1}{F(\bar{h}, \bar{H}, L, \bar{k1}, N)} \quad \ldots(23)$$

C) The Time of Approach

The time of approach is given by:

$$t = \int_{h_o}^{h} \frac{dh}{V} \quad \ldots(24)$$

The time of approach in non-dimensional form for a particular load W may be obtained as:-

$$T = \frac{2\, W\, t\, h_o^2}{\mu B a^3} = \frac{1}{6} \int_{\bar{h}}^{1} \frac{d\bar{h}}{F(\bar{h}, \bar{H}, L, \bar{k1}, N)} \quad \ldots(25)$$

For a non-porous configuration, the non-dimensional pressure distribution, the load carrying capacity and the time of approach can be obtained by substituting $\bar{k1} = 0$ in equations (21), (23) and (25). Similar expressions are obtained by Prakash and Sinha [14] for non-porous systems.

4. RESULTS AND DISCUSSION

For numerical computation, the following values for k1, k2, k3, H1, H2, and H3, based on previous data [11], for articular cartilage, are taken:

$k1/\mu = 3\times10^{-13}\ cm^4/dyne.sec.$, $k2/\mu = 6\times10^{-13}\ cm^4/dyne.sec.$, $k3/\mu = 9\times10^{-13}$

H1 = 200 microne, H2 = 0.2 cm, H3 = 0.28 cm and $\mu = 1$ poise

The value of minimum film thickness h_o is taken as 10^{-3} cm [13] which is in accordance with hydrodynamic lubrication theory. Thus, k1 is equal to 0.3×10^{-6}. The effect of of some parameters has been presented but the values of P, W and T can easily be obtained from equations (21), (23) and (25) for other values.

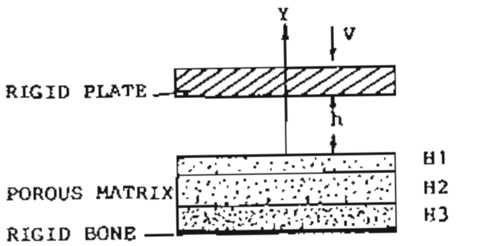


Fig. 1. Articular cartilage model

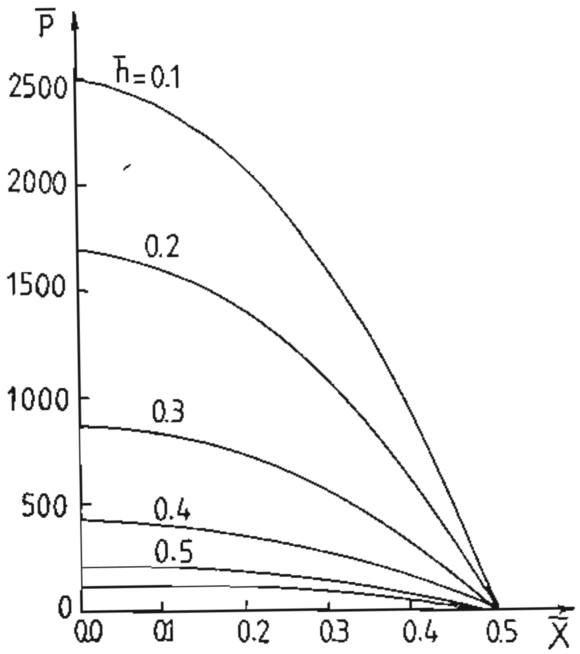


Fig. 2. Non-dimensional axial pressure distribution for different film thickness. ($\bar{K}1=0.3\times10^{-6}$, $N^2=0.7$, $L=6.0$)

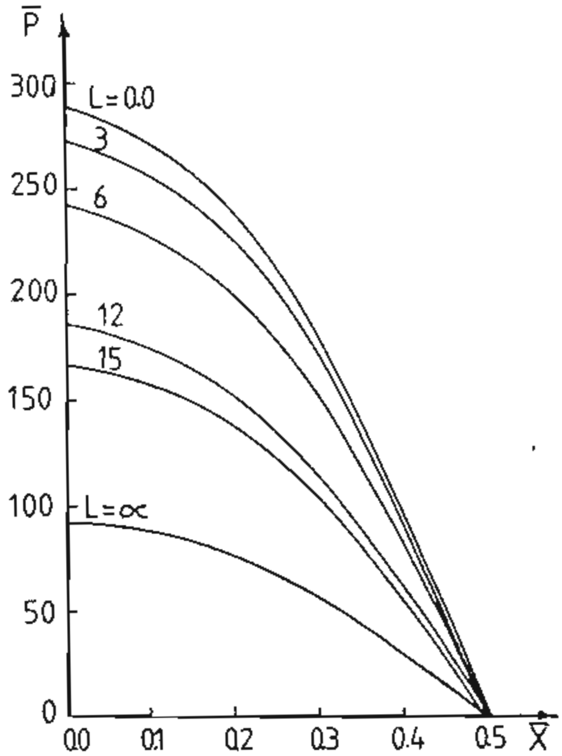


Fig. 3. Non-dimensional axial pressure for different values of shape and size index L. ($\bar{K}1=0.3\times10^{-6}$, $N^2=0.7$, $\bar{h}=0.5$).

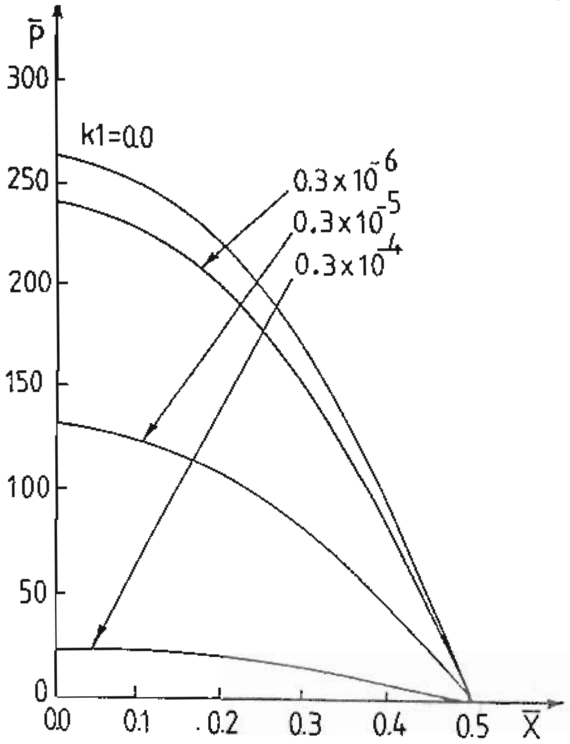


Fig. 4. Non-dimensional axial pressure for different values of porosity of STZ. (N^2=0.7, L=6.0, $\bar{h}$=0.5).

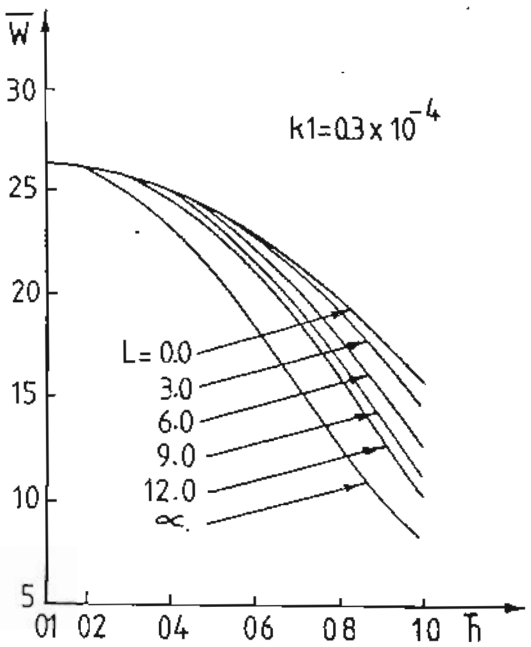
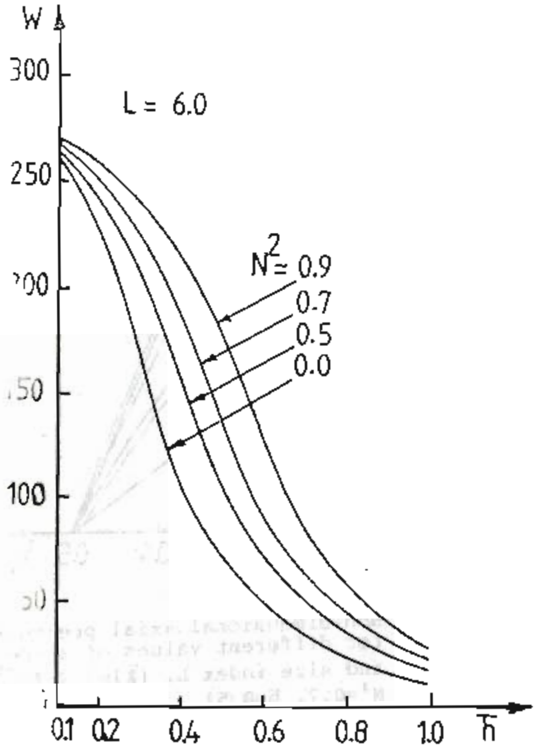


Fig. 5. Variation of load carrying capacity with film thickness for different values of L.



. . 6. Variation of load carrying capacity with film thickness for different values of concentration index N.

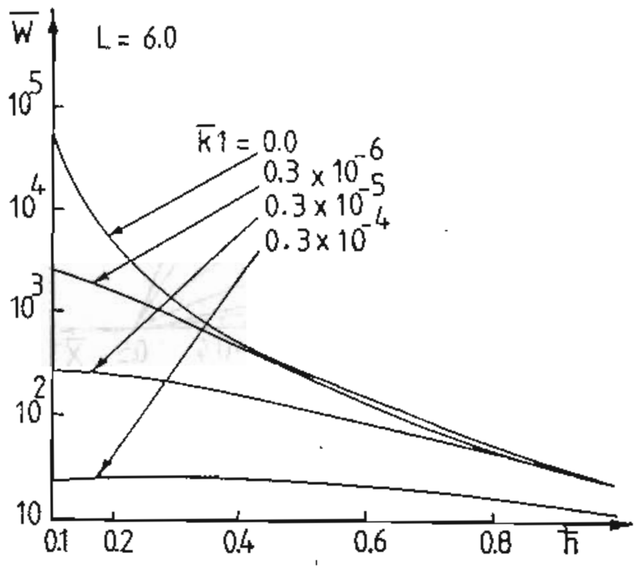


Fig. 7. Variation of non-dimensional load carrying capacity with film thickness for different values of porosity parameter $\bar{k}1$. (L=6.0, N^2=0.7).

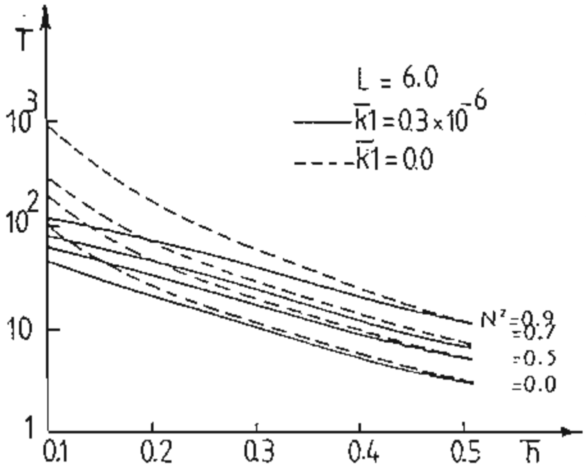


Fig. 8. Variation of time of approach with film thickness for different values of concentration index N.

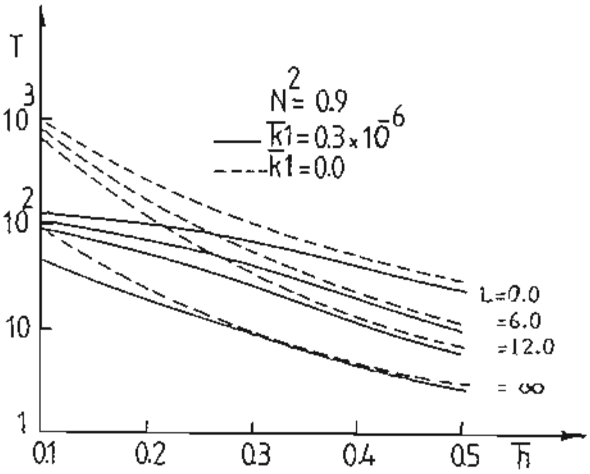


Fig. 9. Variation of time of approach with film thickness for different values of shape and size parameter L.

In Figs. 2 and 3 the variation of non-dimensional axial pressure is shown for different values of film thickness, shape and size index. The results in Fig. 2 are supporting the fact that the decease in film thickness increases the pressure. In Fig. 3, axial pressure displays a decrease with the increase in L. In addition, in the same Fig. 3, $L \to \infty$ corresponds to a Newtonian fluid, which confirms previous findings that the non-Newtonian character of synovial fluid is responsible for higher pressure build-up and load-carrying capacity.

Fig. 4 demonstrates the effect of the porosity of the superfacial tangential zone upon the non-dimensional pressure distribution $\bar{P}$. It is evident that porosity decreases the pressure. The pressure build-up declines very quickly as the porosity increases.

In Fig. 5, it can be seen that the load-carrying capacity decreases as L increases and it is least when the fluid becomes Newtonian, i.e. $L \to \infty$.

Fig. 6 depicts the increase in load-carrying capacity with the increase in the concentration index of articular suspension and it is minimum for Newtonian fluid ($N^2 = 0$). The influence of the porosity of the superfacial tangential zone of cartilage upon the load-carrying capacity is shown in Fig. 7. It is clear that the porosity decreases the load-carrying capacity. Maximum values for load-carrying capacity were obtained for non-porous configuration.

From Figs. 8 and 9 it is seen that the time of approach increases as N increases and L decreases. By comparing the cases of porous and non-porous situations, it can be noted that the time of approach is greater for the non-porous case, which is obvious in view of the fact that the load-carrying capacity and the pressure are smaller in the porous case. Furthermore, the calculated closure time increases as the gap (h) decreases which in turn increases the concentration of the suspended particles.

In general, the present results are in agreement with existing evidences and support the squeeze film lubrication mode as an important aspect in the lubrication of articulating synovial human joints.

5. CONCLUSIONS

From the present theoretical study the following conclusions can be drawn:

1. The decrease in film thickness of synovial fluid in the joint results in an increase in the pressure generated in the fluid. The non-Newtonian character of synovial fluid is the main cause for the build-up of high fluid pressure and load-carrying capacity.

2. As the porosity increases in the superfacial zone of articular cartilage, the pressure declines very quickly.

3. The porosity decreases the load-carrying capacity which is a maximum for non-porous configuration.

4. The closure time of the joint gap increases as the synovial film thickness decreases which produced an increase in the concentration of the suspended particles of the synovial fluid.

5. The results indicate that squeeze film lubrication in articulating synovial joints is an essential feature in the lubrication of these joints.

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