# Kinematics Analysis of a parallel robot with 3 DOF and 4 segments in pure translation

تحليل حركية روبوت متوازي ذو ثلاث درجات من الحرية

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**ملخص**

في هذا المقال يتم دراسة مسألتي الحركة المباشرة و الحركة العكسية لربوت متوازي UPS ذو ثلاث درجات من الحرية و وصلة مركزية لإعطاء الروبوت حركة انزلاقيةPPP .
نوضح كذلك العلاقة بين السرعة في مجال العمل للروبوت و السرعة في المفاصل ، و عكس ذلك، أي المرور من السرعة في المفاصل و الوصول إلى سرعة أداة عمل الروبوت. هذه الدراسة يرا د منها وضع برمجة التحكم في روبوت متوازي.
تم انجاز في المخبر ، نموذج لروبوت متوازي للتأكد من صحة الأنماط المستعملة.

**الكلمات المفتاحية:** روبوت متوازي ، الحركة المباشرة، الحركة العكسية ، وصلة مركزية.

**Abstract**

In this paper we present the direct and inverse analysis of a geometrical model of an UPS manipulator with three degrees of freedom added to a PPP passive central segment. This structure provides a pure translation motion. We will also determine the relations between generalized and articular velocities by using the inverse Jacobian matrix. Further, we determine the reciprocal relations between cartesian and angular velocities of the end-effector via articular velocities by direct re-inversion of the Jacobian matrix. This study aims of implementing the control of a parallel robot manipulator. A prototype of a parallel robot has been built up in our laboratory in order to validate the proposed models.

**Key Words:** direct geometrical model, inverse geometrical model, UPS manipulator, passive central segment, Jacobian matrix, parallel robot.

## 1. Introduction

These last years, the manipulators with a parallel structure have constituted a pole of interest for researchers and industrialists because of their interesting performances: high load capacity, movements at high rate, high mechanical rigidity, low mobile mass, simplicity of mechanical engineering and possibility of locating the actuators and the sensors on the base, or close to the base.

Historically, these mechanical systems were initially developed for applications other than the pure robotics field. The first mechanical system of this type was the machine built by Gough [13] in 1949 to test the tires of the planes. Thereafter, Stewart [2]

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developed this structure to use it as a flight simulator. However, the idea to use parallel structures in robotics is in fact more recent. Indeed, Tidal [3] has suggested this architecture for a machine tool. Hunt [12] has identified certain parallel architectures that offer more significant potential in robotics. In this way, several researchers such as Merlet [1] and Reboulet [3] have deeply analysed these architectures.

Since many industrial applications require only one pure translation movement such as the positioning of a tool in an assembly line, some researchers were therefore interested in the analysis of parallel robots with three degrees of freedom in pure translation.

In this course, Tsai [4] in 1996 has studied the kinematics of a robot with parallel structure of UPU (universal-prismatic-universal) type.

In 2003 Kim and Tsai [6] have analysed a parallel structure of type RPS. Joshi [11] has studied a parallel structure with three degrees of freedom and four bars. Kim [5], Karricato [7, 8], Stock [9] and Callergari [10] have developed the kinematics of a parallel manipulator of 3 d.o.f in pure translation.

In the present paper, we will present the geometrical and kinematics modelling of a parallel structure of three degrees of freedom ( d.o.f ) and four segments with one passive central segment of type Prismatic–prismatic- prismatic ( PPP ) to constrain the structure to move in a pure translation. Analytical solutions have been found and will be presented for the direct and inverse geometrical models as well as for the kinematics model. In order to validate the theoretical analysis, an experimental prototype was built up in our laboratory. The geometrical model has been implemented and used for robot control.

## 2. Geometrical analysis

### 2.1) Kinematics structure

The manipulator under consideration is of three degrees of freedom. It consists of a mobile platform connected to a fixed platform via three active segments and one passive central segment. One calls segment each open chain that connects the base to the mobile platform (Fig.1).

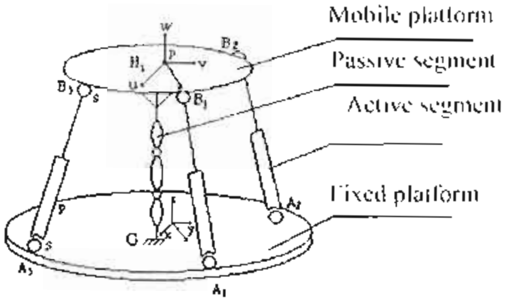


*Fig.1 .a Parallel Robot with four segments*

In order to specify the final structure of our parallel robot, we must, initially, determine the type of the articulations that one must place in each end of the segments. For this purpose, it is necessary to consider the situation where the system is in balance. This means that the mobility of the mechanism stands null when the actuators are blocked in a given configuration.

To determine the type of the articulations, one needs to determine the mobility of an isostatic space mechanism according to the Grübler's formula [1].

$$m = 6(s - n - 1) + \sum_{i=1}^{n} d_i \qquad (1)$$

With

s: the total number of solids including the base

n: the total number of articulations

$d_i$: the number of degrees of freedom of articulation i

In the case of a parallel structure, if one notes

p: the number of segments

And

Ck: the number of degrees of freedom associated with each segment

We have:

$$\sum_{i=1}^{p} Ck = m + 6(p-1) \qquad (2)$$

Since our system has three degrees of freedom and four segments, thus m = 3 and p = 4. By replacing these values into relation (2), we get the following expression:

$$C1 + C2 + C3 + C4 = 21$$

To respect this equation, we can combine different types of Ck

For instance, we can choose C1=3 and C2=C3=C4=6. This means that we have 3 degrees of freedom for the passive segment and an orthogonal kneecap slide and two pivots for the other segments.

Note that we can add additional degrees of freedom that do not modify the movement of the mobile platform.

In our specific case, we consider that the three active segments are connected to the fixed platform with universal link and to the mobile platform with 3 dof kneecaps. The passive central bar has 3 d.o.f and can have several configurations or type of mechanical structures but we choose a structure of the type PPP to give to the manipulator a pure translation. There is thus C1=3, C2=C3=C4=2+1+3 and m=3.

**2.2) Geometrical analysis**

Let us consider a reference frame X, Y, Z of center O related to the fixed platform and a reference frame U, V, W of centre P related to the mobile platform (Fig. 2).

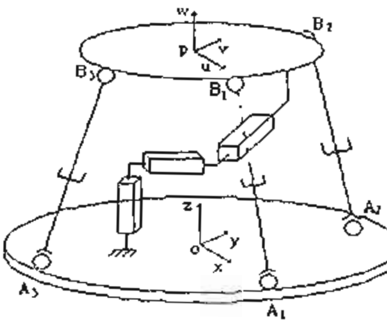


Fig.2. Structure of the passive central segment

The active segments are related to the fixed platform at points $A_1$, $A_2$, $A_3$ which are located at a distance $r_a$ with respect to the centre O and connected to the mobile platform at points $B_1$, $B_2$ and $B_3$. The angle $\beta_i$ is measured between the X axis and the line $OA_i$ and in the same way between the Y axis and the line $PB_i$

In our case: the angle $\beta_1 = 0$.

The co-ordinates of the vectors $OA_i$ and $PB_i$ can be written in the following form

$$^{A}\overline{OA_i} = {}^{A}a_i = [a_{ix}, a_{iy}, 0]^t \qquad (3)$$

$$^{B}\overline{PB_i} = {}^{B}b_i = [b_{iu}, b_{iv}, 0]^t \qquad (4)$$

Where

$$^{A}a_i = [\, ra\, C\beta_i,\ ra\, S\beta_i,\ 0\,]^t,\ i = 1, 2, 3$$

$$^{B}b_i = [\, rb\, C\beta_i,\ rb\, S\beta_i,\ 0\,]^t,\ i = 1, 2, 3$$

Where $C\beta_i$ and $S\beta_i$ are the cosine and sine of the angles $\beta_i$.

Let us consider the point of connection G of the passive segment with the fixed platform and the point of connection H of the same segment with the mobile platform. The co-ordinates of these points of connection are given by:

$$^{A}\overline{OG} = {}^{A}g = [g_x, g_y, 0]^t \qquad (5)$$

$$^{B}\overline{PH} = {}^{B}h = [h_u, h_v, 0]^t \qquad (6)$$

where Px, Py, Pz represents the co-ordinates of the point P which is the centre of the frame U, V, W, with respect to the basic reference frame.

Consider the Vector $^{A}P$ which is used to define the position of mobile platform.

$$^{A}P = \overline{OP} = [\, P_x, P_y, P_z\,]^t \qquad (7)$$

The rotation matrix $^{A}R_B$ is used to define the orientations of the mobile platform with respect to the basic reference frame.

If one notes $\Phi$, $\theta$ and $\psi$ Cardan angles or RTL (Roulis-tangage-lacet) defined by the succession of three rotations around three axes X, Y and Z to pass from the frame $R_0$ to $R_1$

The matrix R is thus written:

$$^{A}R_B = R_z(\phi)\cdot R_y(\theta)\cdot R_x(\psi) \qquad (8)$$

To simplify the writing of the matrix of rotation, we replace in all that follows cosine of the angles by C and the sine by S, then one will have

$$^{A}R_B = \begin{bmatrix} C\phi C_\theta & C\phi S_\theta S\psi - S\phi C\psi & C\phi S_\theta C\psi + S\phi S\psi \\ S\phi C_\theta & S\phi S_\theta S\psi + C\phi C\psi & S\phi S_\theta C\psi - C\phi S\psi \\ -S_\theta & C\theta S\psi & C\theta C\psi \end{bmatrix} \qquad (9)$$

We introduce the matrix $^{A}T_B$ that defines the orientation and the position of mobile platform with respect to the basic reference frame.

$$^{A}T_B = \begin{bmatrix} ^{A}R_B & ^{A}P \\ 0 & 1 \end{bmatrix} \qquad (10)$$

The development of (10) provides an equation system containing six variables ($P_X$, $P_y$, $P_Z$, $\psi$, $\theta$, $\Phi$) where three only are independent.

Let's consider the representation of the point P as the end position of the passive segment with respect to the basic reference frame. The central segment is analysed as an open articulated mechanical system. By utilizing the parameters of denavit-hartenberg, the passage from the mobile platform to the fixed platform is given by:

$$^{A}T_B = {}^{A}T_0\, {}^{0}T_1(\theta_1)\, {}^{1}T_2(\theta_2)\, {}^{2}T_3(\theta_3)\, {}^{3}T_B \quad (11)$$

Since at the point P, the active and the passive segments meet, therefore, relations (10) and (11) can be equalised providing the following closing equation:

$$^{A}T_0\, {}^{0}T_1(\theta_1)\, {}^{1}T_2(\theta_2)\, {}^{2}T_3(\theta_3)\, {}^{3}T_B = \begin{bmatrix} ^{A}R_B & ^{A}P \\ 0 & 1 \end{bmatrix} \quad (12)$$

The member of left-hand side of the equation (12) depends on the articular variables of the passive segment. The member of right-hand side depends on the 6 operational variables $P_X$, $P_Y$, $P_Z$, $\psi$, $\theta$, $\Phi$. The resolution of the equation system that results from (12) enables to determine these 6 operational variables with respect to the 3 articular variables of the passive segment.

### 3. Inverse geometrical model

Each active segment is made of two bodies in slide connection bound by kneecaps to the base and to the mobile platform (fig.2).

The following elements are defined:

Ai: articulation of bar i attached to the base.

Bi: articulation of bar i attached to the mobile platform.

By considering the closing equation, we can obtain the expression of the articular co-ordinates $q_i$ which represent the lengths of the active segments, with respect to the operational co-ordinates.

The geometrical relation is,

$$\overline{OP} = \overline{OAi} + \overline{Ai\,Bi} + \overline{Bi\,P}$$

$$^{A}P - {}^{A}ai - qi + \overline{PBi} = 0$$

After development, we have:

$$q_i = {}^{A}P + {}^{A}b_i - {}^{A}a_i$$

$$q_i = {}^{A}P + {}^{A}R_B\, {}^{B}b_i - {}^{A}a_i$$

$$q_i^2 = [{}^{A}P + {}^{A}R_B\, {}^{B}b_i - {}^{A}a_i]^T \cdot [{}^{A}P + {}^{A}R_B\, {}^{B}b_i - {}^{A}a_i]. \quad (13)$$

We can observe that relation (13) contains a transformation matrix that can be written for a pure translation such as follows:

$$^{A}T_B = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & L_2 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

With: $L_i$ the length of the passive segment for i = 1, 2, 3.

The resolution of (12) by identification is obvious in our particular case:

$$\begin{cases} P_x = L_1 \\ P_y = L_2 \\ P_z = L_3 \end{cases}$$

By developing the equation (13), we have the follow in relation:

$$q_i^2 = \left[\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} - \begin{bmatrix} ra\,C\beta_i \\ ra\,S\beta_i \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} rb\,C\beta_i \\ rb\,S\beta_i \\ 0 \end{bmatrix}\right] \cdot \left[\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} - \begin{bmatrix} ra\,C\beta_i \\ ra\,S\beta_i \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} rb\,C\beta_i \\ rb\,S\beta_i \\ 0 \end{bmatrix}\right]$$

After development, we get:

$$q_i^2 = P_x^2 + P_Y^2 + P_z^2 + r_a^2 + r_b^2 - 2r_ar_b - 2P_xr_aC\beta_i + 2P_xr_bC\beta_i - 2P_yr_aS\beta_i + 2P_yr_bS\beta_i \quad (14)$$

or

$$\begin{cases} q_1^2 = P_x^2 + P_y^2 + P_z^2 + r_a^2 + r_b^2 - 2r_a r_b - 2P_x r_a + 2P_x r_b \\ q_2^2 = P_x^2 + P_y^2 + P_z^2 + r_a^2 + r_b^2 - 2r_a r_b - 2P_x r_aC\beta_2 \\ \quad + 2P_x r_bC\beta_2 - 2P_y r_aS\beta_2 + 2P_y r_bS\beta_2 \\ q_3^2 = P_x^2 + P_y^2 + P_z^2 + r_a^2 + r_b^2 - 2r_a r_b - 2P_x r_aC\beta_3 \\ \quad + 2P_x r_bC\beta_3 - 2P_y r_aS\beta_3 + 2P_y r_bS\beta_3 \end{cases}$$

These relations show that the lengths of active segments can be calculated providing the geometrical characteristics of the robot and the position that have to be reached by the end-effector.

By using the following change of variables,

$e_1 = q_1^2 - q_2^2$

$e_2 = q_1^2 - q_3^2$

$e_3 = q_2^2 - q_3^2$

$e_4 = 2r_aS\beta_2 - 2r_bS\beta_2$

$e_5 = 2r_aS\beta_3 - 2r_bS\beta_3$

$e_6 = -2r_a + 2r_b$

$e_7 = 2\,r_aC\beta_2 - 2\,r_bC\beta_2$

$e_8 = 2\,r_aC\beta_3 - 2\,r_bC\beta_3$

$e_9 = r_a^2 + r_b^2 - 2r_a r_b$

By means of these new variables, we can express the qi as follows:

$$\begin{cases} q_1^2 = P_x^2 + P_y^2 + P_z^2 + e_9 + P_x e_6 \\ q_2^2 = P_x^2 + P_y^2 + P_z^2 + e_9 - P_x.e_7 - P_y.e_4 \\ q_3^2 = P_x^2 + P_y^2 + P_z^2 + e_9 - P_x.e_8 - P_y.e_5 \end{cases} \quad (15)$$

Thus, being given geometrical dimensions of the parallel robot, one can determine the lengths of the active segments which are necessary to move the mobile platform towards the point of the space located at the co-ordinates (Px,Py,Pz)[1].

These expressions (15) enable us to implement the control of the robot based on the inverse geometrical model.

### 4. Direct geometrical model

The direct geometrical model is used to pass from the articular co-ordinates to the operational coordinates expressed in the reference frame of the base.

By combining expressions (15), we can have:

$$q_1^2 - q_2^2 = P_x(e_6 + e_7) + P_y.e_4 \quad (16).$$

$$q_1^2 - q_3^2 = P_x(e_6 + e_8) + P_y.e_5 \quad (17)$$

The resolution of this system aims to determine $P_x$, $P_y$, $P_z$ as follows:

$$\begin{cases} P_x = \dfrac{e_2.e_4 - e_5 e_1}{e_4(e_6 + e_8) - e_5(e_6 + e_7)} \\ P_y = \dfrac{e_1 - P_x(e_6 + e_7)}{e_4} \\ P_z = \pm\sqrt{q_1^2 - P_x^2 - P_y^2 - e_9 - P_x e_6} \end{cases} \quad (18)$$

Relations (18) allow the passage from the articular co-ordinates to the configurations of the final body as well as the implementation of the control of the robot based on the direct geometrical model.

## 5. Inverse kinematics model

The inverse model establishes a relation between the articular velocity and the operational velocity. As a consequence, the inverse Jacobian matrix establishes a relationship between Cartesian and angular velocities and articular velocities.

The fundamental equations of inverse kinematics directly express the articular variables according to the orientation and the position parameters of the mobile platform. By simple derivation of (15) one can deduce from it the inverse Jacobian matrix for any representation of the orientation. The inverse jacobian matrix is given analytically by the relation:

$$\frac{dq}{dt} = J^{-1}\frac{dx}{dt}$$

Where

$$J^{-1} = \begin{bmatrix} \frac{\partial q_1}{\partial x} & \frac{\partial q_1}{\partial y} & \frac{\partial q_1}{\partial z} \\ \frac{\partial q_2}{\partial x} & \frac{\partial q_2}{\partial y} & \frac{\partial q_2}{\partial z} \\ \frac{\partial q_3}{\partial x} & \frac{\partial q_3}{\partial y} & \frac{\partial q_3}{\partial z} \end{bmatrix}$$ from which

$$J^{-1} = \begin{bmatrix} \frac{2P_x+e_6}{2q_1} & \frac{2P_y}{2q_1} & \frac{2P_z}{2q_1} \\ \frac{2P_x-e_7}{2q_2} & \frac{2P_y-e_4}{2q_2} & \frac{2P_z}{2q_2} \\ \frac{2P_x-e_8}{2q_3} & \frac{2P_y-e_9}{2q_3} & \frac{2P_z}{2q_3} \end{bmatrix} \quad (19)$$

Thus, the inverse Jacobian matrix can be used for the study of the singular positions of the parallel manipulator, for the evaluation of its maneuverability and also for the optimization of its architecture.

## 6. Direct kinematics model

the calculation of the inverse jacobian matrix is generally easy, the expressions obtained are however complex and obtaining the direct jacobian matrix by inversion, like the method used by Mr. Stock and K. Miller [ 9 ], is a difficult spot.

For our manipulator UPS, by simple derivation of the expressions (14) we obtain the direct jacobian matrix which is written.

$$J = \begin{bmatrix} \frac{2q_1}{e_6+e_7} & \frac{-q_2}{e_6+e_7} & \frac{-q_3}{e_6+e_7} \\ \frac{2q_1(p_x(e_6+e_7)+e_3)}{e_4} & \frac{q_2(2p_x(e_6+e_7)-e_3)}{e_4} & \frac{-q_3 e_3}{e_4} \\ \frac{q_1}{p_z}\left[1+\frac{2p_x-e_5}{e_6+e_7}+\frac{2p_x\left(p_x(e_6+e_7)+e_3\right)}{e_4}\right] & \frac{q_2}{p_z}\left[\left[\frac{-p_x}{e_6+e_7}+\frac{p_x\left(2p_x(e_6+e_7)-e_3\right)}{e_4}\right]+\frac{2e_5}{e_6+e_7}\right] & \frac{q_3}{p_z}\left[\frac{-p_x}{e_6+e_7}-\frac{p_x}{e_4}+\frac{2e_5}{e_6+e_7}\right] \end{bmatrix} \quad (20)$$

## 7. Experimental prototype and numerical results

The experimental prototype of the parallel robot which has been in our laboratory is constituted of tack welded structure and D.C. motors as actuators (Fig. 3 and Fig.4). A control based on the direct geometrical model as well as a control based on the inverse geometrical model have been implemented. We also have developed a graphical user interface which allows the user to specify the articular or the Cartesian co-ordinates.

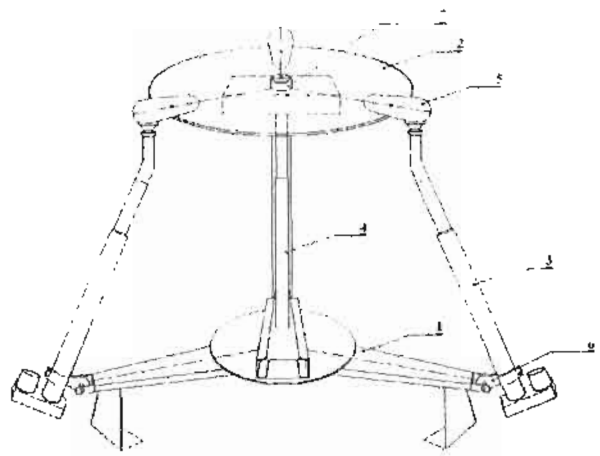

Fig 3. Scheme of our parallel robot prototype

*1.fixed Base 2 Mobile Platform.3. Actuator.*

*4. Passive segment 5 Kneecap. 6. Ball and socket joint*

*7 Place of the final body*

### 7.1) Geometrical characteristics

The prototype built up in our laboratory has the following geometrical data (see Fig.3 and Fig.4 ) :

ra = 690 mm, rb = 350 mm, $\beta_1$= 0° . $\beta_2$= 120° et $\beta_3$= 240°

The points of connection of the segments with the fixed platform are :

$A_1$=[690, 0, 0], $A_2$=[-345, 597.55, 0] et $A_3$=[-345, -597.55, 0 ]

The points of connection of the segments with the mobile platform are:

$B_1$=[350, 0, 0] , $B_2$=[-175, 303.1, 0] et $B_3$=[-175, -303.1, 0]

### 7.2) Simulation Results

In order to verify the obtained relations of both direct and inverse kinematics; we have performed a simulation by implementing the code corresponding to these relations. We provide the coordinates of some points in the attainable work space of our robot as inputs and we check the results which are articular coordinates. Conversely, we introduce these results (articular coordinates) as inputs and we except to obtain the operational coordinates. Results are shown in Table 1.

| Inverse Geometry | | direct Geometry | |
|---|---|---|---|
| Px = 100<br>Py = 50<br>Pz = 600 | $q_1$= 648.1512<br>$q_2$=701.8366<br>$q_3$=742.7205 | $q_1$= 648.1512<br>$q_2$=701.8366<br>$q_3$=742.7205 | Px = 100<br>Py = 50<br>Pz = 600 |
| Px = 120<br>Py = 10<br>Pz =710 | $q_1$= 743.3707<br>$q_2$=817.9440<br>$q_3$=825.2465 | $q_1$= 743.3707<br>$q_2$=817.9440<br>$q_3$=825.2465 | Px = 120<br>Py = 10<br>Pz =710 |
| Px = 15<br>Py = 20<br>Pz =600 | $q_1$= 682.6602<br>$q_2$=685.2229<br>$q_3$=702.2159 | $q_1$= 682.6602<br>$q_2$=685.2229<br>$q_3$=702.2159 | Px = 15<br>Py = 20<br>Pz =600 |
| Px = 50<br>Py = 70<br>Pz =680 | $q_1$= 742.5631<br>$q_2$=749.0799<br>$q_3$=802.2684 | $q_1$= 742.5631<br>$q_2$=749.0799<br>$q_3$=802.2684 | Px = 50<br>Py = 70<br>Pz =680 |
| Px = 0<br>Py = 0<br>Pz =533.9856 | $q_1$= 650<br>$q_2$=650<br>$q_3$=650 | $q_1$= 650<br>$q_2$=650<br>$q_3$=650 | Px = 0<br>Py = 0<br>Pz=533.9856 |

Table 1: numerical results for the direct and inverse geometry

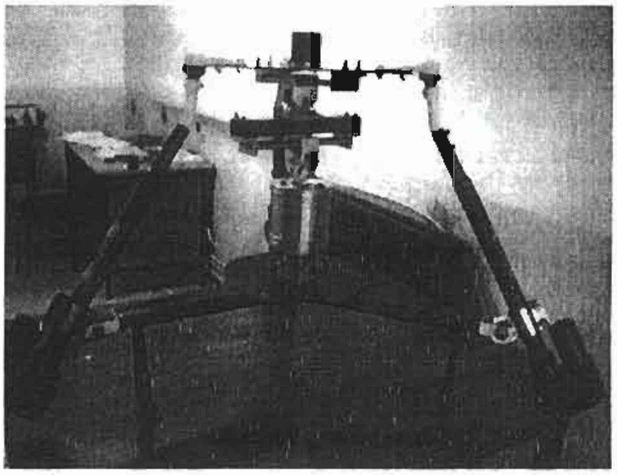

Fig.4. picture of our parallel robot

### 7.3) Comments

From table 1, we can conclude that the theoretical expressions concerning inverse and direct kinematics are well verified. In the practice, our results agree qualitatively with our theorical expressions.

Control based on inverse geometrical has been tested. The experimental results reveal that the robot moves towards the vicinity of the position specified in operational co-ordinates by the user.

In addition, experiments of tele-operation were carried out on site via Intranet where the vision is ensured by a webcam. In these experiments, the operator controls the robot by giving commands (impulses) via a graphical user interface.

## 8. Conclusion

We have launched the analysis of a parallel robot with three degrees of freedom and four segments. We have chosen a particular structure of the passive segment to give a pure translation to the robot. We have obtained analytical solutions for the direct and inverse geometrical model. We have also obtained the jacobian matrix by simple inversion of the inverse jacobian matrix.

Our theoretical expressions concerning inverse and direct kinematics have been well verified thought simulation. Nevertheless, in the practice, our results agree qualitatively with our theoretical expressions.

Indeed, errors are stemming from the fact that our model does not take into account the inertia of the system and the modeling of motors. In the same way, the absence of feedback in this experimental prototype does not enable to compensate these errors. To this end, the kinematics model is in the course of implementation.

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