Menofia University **Faculty of Engineering** **Basic Engineering Sci. Department** **Academic Year : 2019-2020** **Date : 09 / 08 / 2020**	Minoufia University	**Subject: Theory of Probability and statistics** **Code: BES 508** **Time Allowed : 3 hours** **Year : Master** **Total Marks: 100 Marks**

Answer all the following questions: (الامتحان في ثلاث صفحات)

Question 1 **35 marks**

A A box contains **3** balls; one red, one black and one white. A ball is selected randomly from the box, then restored to the box, then another ball is selected randomly, write the outcomes space of this experiment.

B Consider a sample space $\boldsymbol{S}$ and three events $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ For each of the following events draw a Venn diagram representation as well as a set expression.

1) Among $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$, only $\boldsymbol{A}$ occurs.
2) At least one of the events $\boldsymbol{A}$, $\boldsymbol{B}$, or $\boldsymbol{C}$ occurs.
3) $\boldsymbol{A}$ or $\boldsymbol{C}$ occurs, but not $\boldsymbol{B}$.

C I roll a fair die twice and obtain two numbers X_1= result of the first roll, and X_2 =result of the second roll. Find the probability of the following events:

a. $\boldsymbol{A}$ defined as "$X_1 < X_2$ ";
b. $\boldsymbol{B}$ defined as "You observe a **6** at least once".

Question 2 **30 marks**

A In the experiment of tossing a die twice, if A is the event of getting two numbers one of them is more than or equal to 5, and B is the event of getting two members m and n from which $|m-n| = 2$, and C is the event of getting two numbers, one is odd and the other is prime. Find the following events, in each case, draw a Venn diagram, and shade the region representing each one. Then calculate:

(a) $P(A)$	**(d)** $P(B^C)$	**(g)** $P(A-B)$
(b) $P(A^C)$	**(e)** $P(A\cap B)$	**(h)** $P(B-A)$
(c) $P(B)$	**(f)** $P(A\cup B)$	

B If $\boldsymbol{A}$ and $\boldsymbol{B}$ are independent events, prove that:

1) A^c and B^c are *independent events.*
2) A^c and $\boldsymbol{B}$ are *independent events.*
3) $\boldsymbol{A}$ and $B^c$are *independent events.*

C In a certain assembly plant, three machines, $B1$, $B2$, and $B3$, make **30%**, **45%**, and **25%**, respectively, of the products. It is known from past experience that **2%**, **3%**, and **2%** of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected.

(i) What is the probability that it is defective?

(ii) If a product was chosen randomly and found to be *defective*, what is the probability that it was made by machine $B3$?

(iii) If a product selected at random is found to be defective, which machine was most likely used and thus responsible?

D A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building. Find the probability that:

a) Both the ambulance and the fire engine will be available,

b) The ambulance or the fire engine will be available,

c) Only one of the ambulance or the fire engine will be available,

35 marks

Question 3

A Consider a random variable X that is equal to

$$X = \{1.2.3\}$$

If we know that the probabilities as shown in table

x	1	2	3
$P(x)$	0.5	1/3	k

a) Find the probability k.

b) Construct a probability graph.

c) Find the distribution function for the random variable X, and

d) Graph this distribution function.

B Suppose that a pair dice is tossed and let the discrete random variable X denote the sum of the points. Obtain the range of discrete random variable X.

a) Find the probability function corresponding to the random variable X, and

b) Construct a probability table and a probability graph.

d) Find the distribution function for the random variable X, and

e) Graph this distribution function.

f) Find: [i] $P(3 \le x \le 7)$ [ii] $P(x < 4)$

C Let the continuous random variable X denote the diameter of a hole drilled in a sheet metal component. The target diameter is 12.5 millimeters. Most random disturbances to the process result in large diameters. Historical data show that the distribution of X can be modeled by a probability density function:

$$f(x) = 20e^{-20(x-12.5)}, \quad x \ge 12.5$$

If a part with a diameter larger than 12.60 millimeters is scrapped,

a) What proportion of parts is scrapped?