VOLTAGE CONTROLLED-ELECTROOPTIC DIRECTIONAL COUPLER WITH PARALLEL ELECTRODES

التحكم بالجهد للروابط الكهروضونية الموجهة مع الأقطاب المتوازية

A. M. Zaghloul
Lecturer of Electronics and Communications, Faculty of Engineering, Zagazig University

د. عادل زغلول محمود - كلية الهندسة - جامعة الزقازيق

الخلاصة : في هذا الجوار ثم تم تصميم الروابط الكهروضوئية الموجهة بين مادتين متشابهتين أو مادتين مختلفتين مع جهدين للتحكم. تم معالجة المادة الغير متماثلة كمادة متماثلة مع تعديل كلا من سمكها ومعامل انكسارها. تم حساب معامل الربط بخلط طريقة معامل الانكسار الفعال EIM مع نظرية ربط الحالات CMT. البارامترات التي تغير قيمة الفرق بين معاملي الانتشار لدلائل الموجة Δβ تؤثر أيضا في قيمة معامل الربط ولذلك القدرة المنقولة بين دلائل الموجة لا يمكن حسابها كدالة في Δβ فقط. اتضح أن معامل الربط يعتمد علي معامل الانكسار بكل دليل موجة قبل وبعد جهد التحكم. في حالة المواد المتشابهة معامل الربط يزداد بزيادة فرق الجهد بين جهدي التحكم واتضح أنة في هذه الحالة مادة KNbO3 تعطى أكبر معامل ربط وفي هذه الحالة أيضا معامل الربط يزداد مع اختلاف توجيه المحاور للمادتين. ولذلك معامل الربط بين مادتين مختلفتين يزداد كثيرا حسب نوع المادتين. وقد وجد أن أكبر معامل ربط يحدث بين المادتين LiTaO3, و BaTiO5 . ونلاحظ أن معامل الربط بين مادتين مختلفتين يزداد أو يقل مع جهد التحكم علي حسب المادتين. وقد تم دراسة تأثير كل البارامترات علي معامل الربط و اتضح ان النتائج الرقمية كما هو متوقع.

ABSTRACT: An algorithm is presented to find the effect of controller voltages, orientation of the crystallographic axes (Z-cut, Y-cut and X-cut) and types of the two cores electrooptic materials on both coupling coefficient (C) and power transferred between two adjacent rectangular electrooptic waveguides. Coupling coefficient is calculated by mixing effective index method (EIM) with coupled mode theory (CMT). The electrooptic material is treated as isotropic material with very thin metal electrodes. The applied voltage induces a change of refractive index of waveguides cores and so, there are changes of both the coupling coefficient and the difference between propagation constants of the two waveguides (Δβ). Coupling coefficient between two dissimilar electrooptic materials becomes very large than that between two similar electrooptic materials. Maximum coupling coefficient is occurred between $LiTaO_3$ and $BaTiO_3$. Change of the coupling coefficient with the controller voltages depends on the type of the electrooptic material. The coupling coefficient depends upon the optical polarization (E^x_{pq} or E^y_{pq}) and the operating wavelengths (λ).

1- INTRODUCTION

Optical waveguide directional couplers are the building blocks of many integrated optic devices, such as signal dividers, wavelength division multiplexers, and modulators/switches [1-5]. When a voltage is applied as shown in Fig.1, there is a change in the refractive index due to the applied electric field. By implementing an appropriate design [5], phase mismatch (Δβ) is induced in the optical fields of the two waveguides [6]. In conventional couplers, power switching is achieved [1] and the transferred power between the two waveguides are controlled by the applied voltage [7]. By adjusting the device parameters, most of the power can be transferred back to guide 1[5]. The effect of the optical field on the change of refractive index is neglected. The directional coupler is composed of two diffused waveguides of an electrooptic crystal [8] as shown in Fig.1. There are six orientation cases between the crystallographic axes (a, b and c) and the system axes (x, y and z). For some orientation cases, the crystallographic axes must be rotated by an angle (θ) to eliminate the cross products terms of the index ellipsoidal (IE) as indicated in Appendix A. where the applied voltage is changed (in y-direction), the values of rotated angle (θ) becomes changed. Then for voltage controlled electrooptic directional coupler, cases which need an angle θ are neglected.

Accepted August 28, 2005.

Although, the available orientation cases are, cases 1, 2 and 5 (for $LiNbO_3$ and $LiTaO_3$), cases 1 and 2 (for $KNbO_3$, $BaTiO_3$ and $BaTiO_5$). The orientation cases 1 and 2 are similar for uniaxial materials ($LiNbO_3$, $LiTaO_3$, $BaTiO_3$ and $BaTiO_5$) but cases 1 and 2 are dissimilar for biaxial material ($KNbO_3$) as indicated in Appendix A.

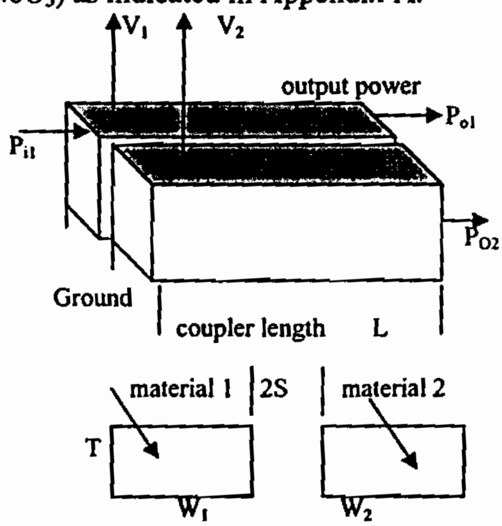


Fig.1 Electrooptic active directional coupler

This study is done with case 1. A study of the polarization dependence in the coupling coefficient of a directional coupler is done, but in this study, E^x_{pq} is used to avoid the propagation losses due to metal electrodes (where, losses of TM mode very large than that losses of TE mode). With very thin metal electrodes, anisotropic waveguide can be treated as isotropic wave guide [9] with modified the waveguide thichness and refractive index. In this study, coupling coefficient between two similar and dissimilar electrooptic materials with different orientations axes is calculated. Effect of applied voltages on C is estimated also,power transferred depends on both C and $\Delta\beta$.

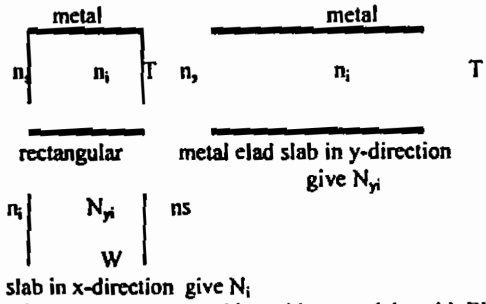


Fig. 2 Rectangular wave guide and its two slabs with EIM

2- MATHEMATICAL ANALYSIS

Coupling coefficient (C) and power transferred between guide 1 (P_1) and guide2 (P_2) are calculated with the following steps:

1- Calculation of the index change due to the applied voltage V, as explained in Appendix A.

2- Conversion the anisotropic core material into equivalent isotropic material using [9,10] modified waveguide thickness (T) and refractive index (n) as:

$n_i=(n_{xi}\, n_{yi})^{0.5}$, $T_{iso}=(\varepsilon_{xi}/\varepsilon_{yi})^{0.5}\, T_{aniso}$ (1)

3- Calculation of the propagation constant of each individual waveguide (β) by using effictive index method (EIM). β is calculated by solving eigen value equation for each slab waveguide (Fig.2) [11];

$\beta_i=k_0\, N_i$ (2)

where, $k_o=2\pi/\lambda$ and N_i is the effective refractive index of guide i and i=1,2.

4- Calculation of the coupling coefficient (C) by using EIM to convert two rectangulars directional coupler into two slabs directional coupler (Fig.3)[1,12]. Coupled mode method used to calculate coupling coefficient between the equivelent two slabs (Appendix B) [13-15].

5- Calculation of power transferred between two lossless rectangular waveguides defined as [16, 17]

$P_2(z)=(C^2/\,C^2_n)\,\sin^2 C_n z$,

$P_1(z)=1-P_2(z)$. (3)

where, $C^2_n=C^2+\Delta\beta^2/4$, $\Delta\beta=\beta_1-\beta_2$

So, transferred power depends upon both C and $\Delta\beta$ not $\Delta\beta$ only, where, values of C depends on the parameters which effect on both β_1 and β_2.

3- RESULTS AND DISCUSSION

The active directional coupler has two types of parameters. The first type is the structure parameters and they are, waveguide thickness (T), width of guide 1 (W_1), width of guide 2 (W_2), distance between the two waveguides (2S), material of guide 1(M_1) material of guide 2 (M_2), orientation case of materials 1 and 2 and the substrate material (n_s). The second type is the operating parameters and they are, operating wavelength (λ), mode numbers in x and y directions (p and q, respectively) , applied voltages (V_1 and V_2) and polarization of the applied optical field (E^x_{pq} or E^y_{pq}). M_i=1 ($LiNbO_3$), M_i=2 ($LiTaO_3$), M_i=3 ($KNbO_3$), M_4=4 ($BaTiO_3$) and M_5=5 ($BaTiO_5$), where, i=1 material in guide 1 and i=2 material in guide 2 . Data of main design example are T=4μm, W_1=W_2=2μm, S=1μm, material of guide 1 ($LiNbO_3$), material of guide 2 ($LiNbO_3$), orientation cases of materials 1 and 2 are case 1, n_s=1.502, λ=0.633μm, p=q=0, V_1=0 and V_2=20 volt and E^x_{00}. Data for electrooptic materials are indicated in Appendix C. Coupling coefficient (C) increased with absolute value of the difference, DN=N_1-N_2 [12] so, C decreased with any parameter which make DN decreased.

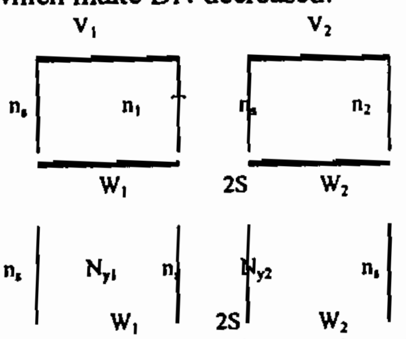


Fig.3 Two rectangular directional coupler and their equivalent two slabs directional coupler.

As suggested [12], C decreased with both waveguide thickness, T, (Table 1), wave-guides width, W_1 and W_2, (Table 2), space between waveguides, S, (Table 3) and refractive index, n_s, (Table 4) but C increased with the operating mode numbers, p and q, (Table 5). Coupling coefficient between the same electrooptic materials (C_{sym} : M_1=M_2) depends on the amplitude of the difference, V_2 -V_1, (Fig.4) and C_{sym} ($KNbO_3$)> C_{sym} ($LiTaO_3$)> C_{sym} ($LiNbO_3$) > C_{sym} ($BaTiO_5$) > C_{sym} ($BaTiO_3$). C_{asym} is decreased or increased with both V_1 and V_2 as shown in Fig.4 (depend upon the values of DN, Appendixs C and D). But the valuc of C_{sym} becomes very small if it is compared with C_{asym} (coupling coefficient between two different electrooptic materials M_1≠M_2) as shown in Figs.4 and 5. Maximum coupling coefficient occurred between $LiTaO_3$ and $BaTiO_5$ (Fig.5). Two similar electrooptic materials act as two different electrooptic materials if the orientation case of material 1 dissimilar than that of material 2, also coupling coefficient between two different materials decreased if the orientation cases of the two materials becomes dissimilar (Table 6). By assume that all other parameters are independent upon wavelength, the coupling coefficient depend upon the operating wavelength (λ) [16] (Fig.6) where, N_1 and N_2 are depend on the operating wavelength. Power transferred (P_2) depend on the applied controlled voltages (Fig.4) where, both C and $\Delta\beta$ depend on the applied voltages. Also P_2 depends on both C and $\Delta\beta$ (Fig.7) where, parameters which change $\Delta\beta$ will variable the value of C (exceptional, with especially very difficult conditions). Dependence of P_2 on the coupler length (L) is shown in Fig. 8, which indicated that P_2 increased until L equal L_{max}, (length of coupler at which the maximum power transferred is occurred) dependence on the values of C and $\Delta\beta$ (i.e. V_2). Polarity of the applied voltage effect on the value of coupling coefficient where, C (V_1=0, V_2=100) =7.131577 nm^{-1}, but C (V_1=0, V_2=-100) =7.131100 nm^{-1}. Also coupling coefficient affected by the polarization of the optical field where, C (V_1=0, V_2=100, E^x_{00}) =662.84 nm^{-1}, but C (V_1=0, V_2=100, E^y_{00}) =663.023 nm^{-1}.

Table 1: Effect of waveguide thickness (T) on C with case 1, M_1=M_2=1, V_1=0, V_2=20 volt, W_1=W_2 =2 μm, S=1μm, n_s=1.502, λ=0.633 μm and E^x_{pq}

T (μm)	2	4	8
C $(nm)^{-1}$	2.860069	1.426220	0.7123947

Table 2: Effect of waveguide widths (W_1 and W_2) on C with case 1, M_1=M_2=1, V_1=0, V_2=20 volt, T=4 μm, S=1 μm, n_s=1.502, λ=0.633μm and E^x_{pq}

W_1 (μm)	2	4	8
W_2 (μm)	2	4	8
C $(nm)^{-1}$	1.426220	1.422405	1.421452

Table 3: Effect of space (S) on C with case 1, M_1=M_2=1, V_1=0, V_2=20 volt, W_1=W_2=2 μm, T=4 μm, n_s=1.502, λ=0.633 μm and E^x_{00}

S (μm)	0.1	0.25	1.0
C $(nm)^{-1}$	2.347446	1.428673	1.426220

Table4:Effect of refractive index (n_s) on C with case 1, M_1=M_2=1, V_1=0, V_2=20 volt, W_1=W_2=2 μm, T=4 μm, S=1 μm, λ=0.633 μm and E^x_{00}

n_s	1.50	1.75	2.00
C $(nm)^{-1}$	1.426220	1.423359	1.416206

Table 5: Effect of mode numbers (p and q) on the coupling coefficient (C nm^{-1}) with case 1, M_1 =M_2=1, V_1=0, V_2=20 volt, W_1=W_2=2 μm, n_s=1.502, T=4 μm, S=1 μm, n_s=1.502, λ=0.633μm and E^x_{pq}

P	0	0	1	1
Q	0	1	0	1
C	1.426220	1.430035	1.443386	1.446724

Table 6: Effect of the orientation cases on the coupling coefficient (C nm^{-1}) with V_1=0 volt, W_1=W_2=2 μm, T=4 μm, S=1 μm, n_s=1.502, λ=0.633 μm and E^x_{pq}

	C for $KNbO_3$ * $KNbO_3$			
V_2 (volt)	Cases 1*1	Cases 2*2	Cases 1*2	Cases 2*1
20	2.296	0.642	77.414	80.352
60	6.892	1.922	76.134	84.948
100	11.487	3.203	74.852	89.542
	C for $KNbO_3$ * $BaTiO_3$			
V_2 (volt)	Cases 1*1	Cases 2*1		
20	724.328	646.274		
60	721.623	643.569		
100	718.919	640.865		

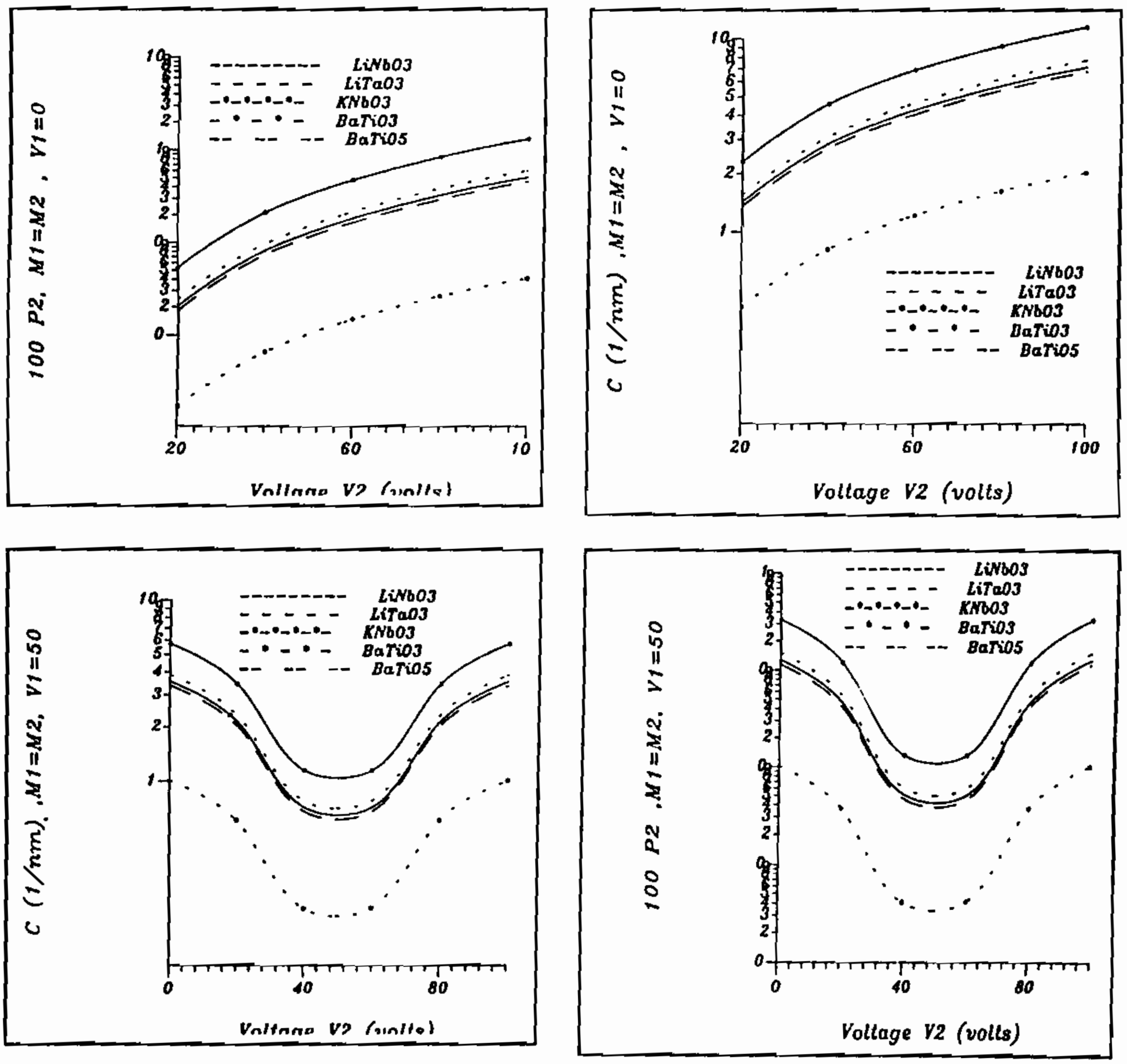


Fig. 4 Coupling coefficient (C) and power transferred (P_2) between two similar waveguides with W_1=W_2=2μm, T=4 μm, S=1 μm, n_s=1.502, λ=0.633 μm, E^x_{00},
Upper Figs. With V_1=0 , lower Figs. V_1=50 volts

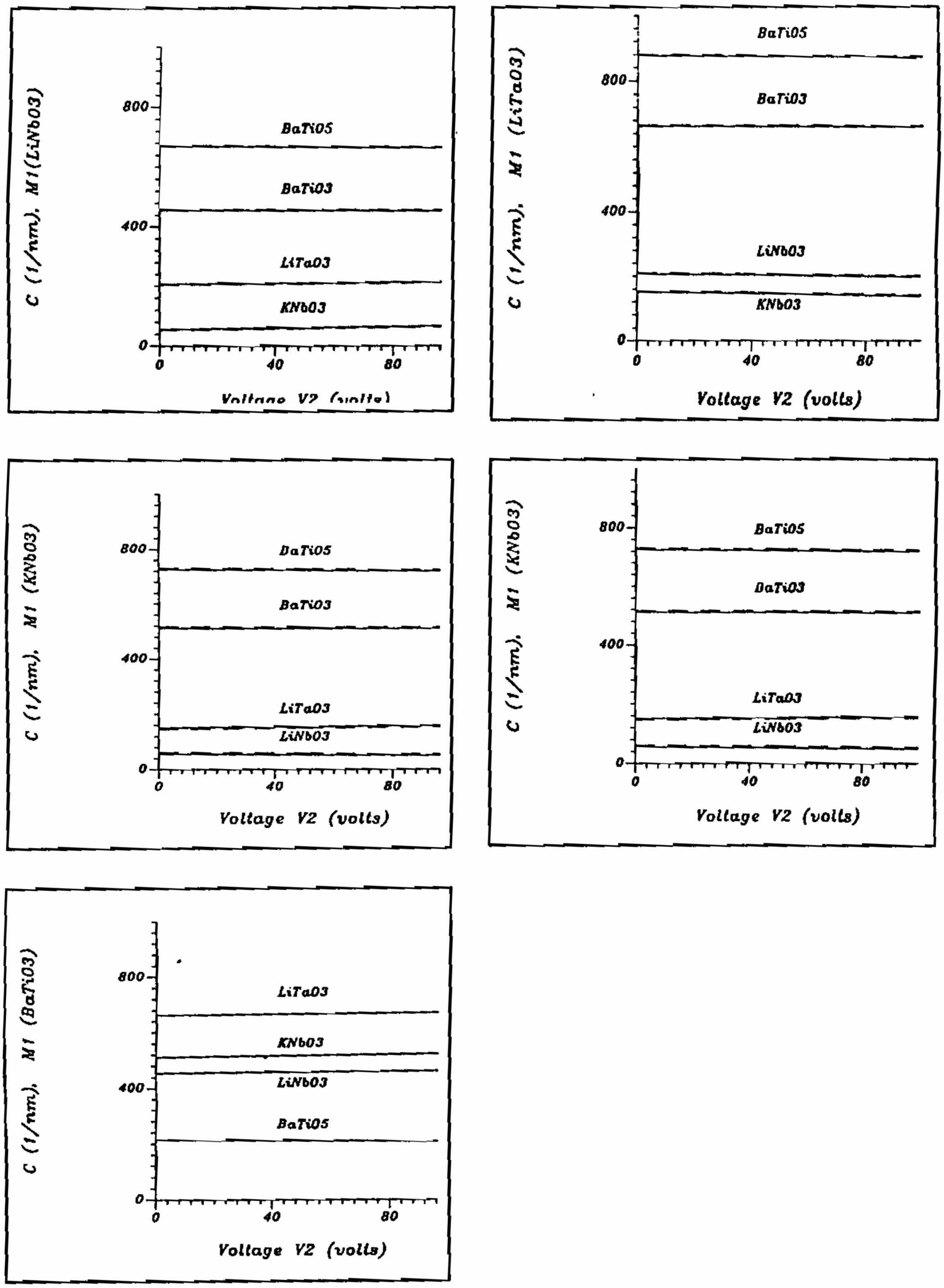
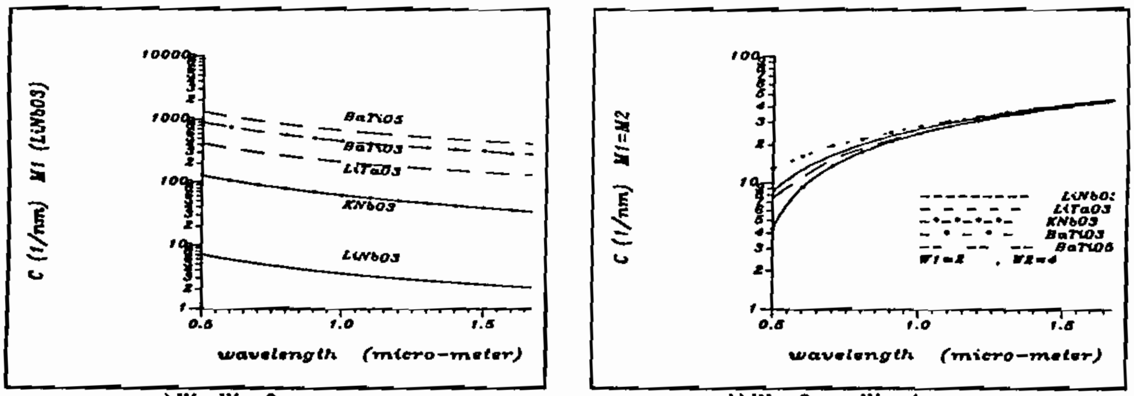


Fig. 5 Effect of the dissimilar electrooptic materials on the coupling coefficient (C) with $W_1=W_2=2\mu m$, $T=4\ \mu m$, $S=1\ \mu m$, $n_s=1.502$, $\lambda=0.633\mu m$, E^x_{00}
Solid lines with $V_1=0$, dashed lines with $V_1=50$ volts



a) $W_1 = W_2 = 2$ μm b) $W_1 = 2$ μm, $W_2 = 4$ μm

Fig. 6: Effect of the operating wavelength (λ) on the coupling coefficient (C) between two similar wave guide, with V_1=0, V_2=20 volts, T=4 μm, S=1 μm, n_s=1.502, E^x_{00}
(all the other parameters are assumed independent upon wavelength)

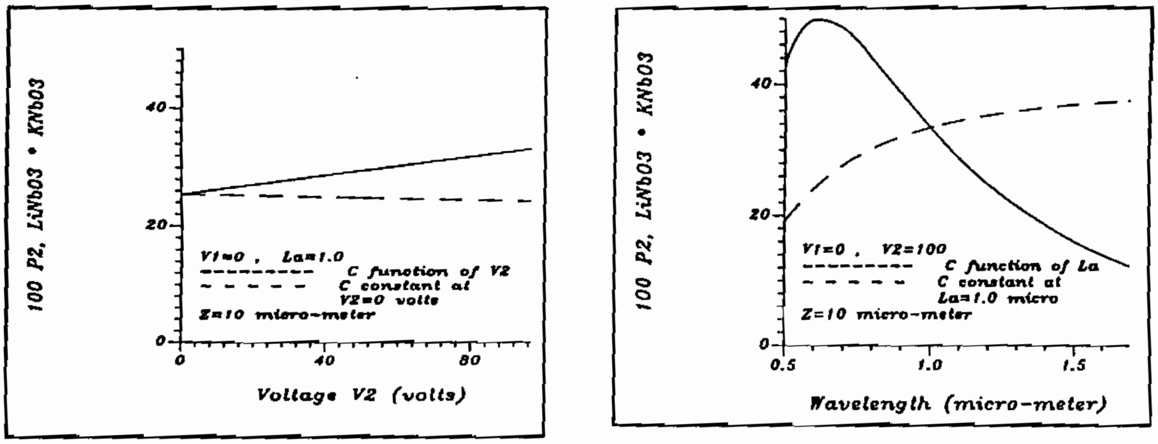


Fig. 7: Comparison between power transferred (P_2) which calculated with:
Solid line, both C and Δβ are variables (function of V_2 or λ),
dashed line, assuming constant coupling coefficient (C) and variable Δβ (function of V_2 or λ),
$W_1 = W_2 = 2$ μm, T=4 μm, S=1 μm, n_s=1.502, E^x_{00},
(all the above parameters are assumed independent upon wavelength)

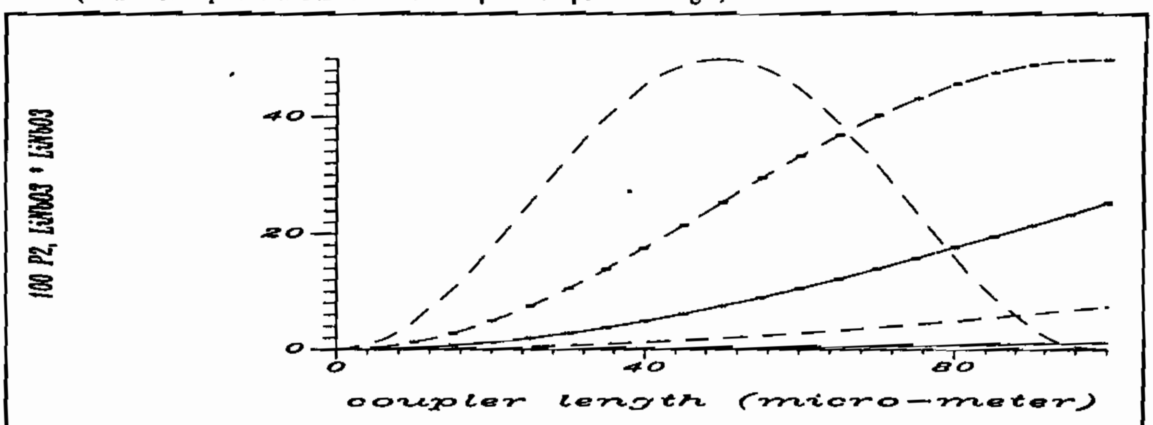


Fig. 8: P_2 as a function of coupler length L at different values of V_2 (i.e. C)
with, V_1=0, $W_1 = W_2 = 2$ μm, T=4 μm, S=1 μm, n_s=1.502, E^x_{00},
Solid (V_2=10, i.e. C=1.124382 nm^{-1}), dashed (V_2=25, i.e. C=2.810478 nm^{-1}),
Solid with samples (V_2=50, i.e. C=5.619049 nm^{-1}), dashed with samples (V_2=100, i.e. C=11.239050 nm^{-1}),
Large dashed (V_2=200, i.e. C=22.479060 nm^{-1})

SUMMARY

In this algorithm, an electrooptic directional coupler is designed using two similar or dissimilar materials with two controller applied voltages. Anaisotropic materials are treated as an isotropic material with modified both thickness and refractive index of materials. Coupling coefficient (C) is calculated by mixing both effective index method and coupled mode theory. Parameters which change the difference between two propagation constants of the individual waveguides ($\Delta\beta$), also, effect on the value of coupling coefficient and so, power transferred between two waveguides can not be calculated as a function of $\Delta\beta$ only.

The coupling coefficient depends on the value of the modified refractive index of each material before and after the controller voltages (V_1 and V_2). In case of two similar materials, coupling increased with the difference controller voltages (ΔV) and KNbO3 is the best material. Coupling becomes more if the orientation axes of both similar materials are different. And so, coupling becomes very large with two dissimilar materials. The best pair of materials are $LiTaO_3$ with $BaTiO_5$. Coupling between pair of dissimilar materials increased or decreased with the controller voltages.

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Appendix A: Change of refractive index [18-21]

When an electric field applied upon electrooptic (EO) material, a new index ellipsoidal (IE) shape occurred, where there are changed in both scale and orientation from original one. The new IE with linear EO effect (Pockels effect)is [19]

$$(n_a^{-2}+r_{11}E_a+r_{12}E_b+r_{13}E_c)\,a^2 + (n_b^{-2}+r_{21}E_a+r_{22}E_b+r_{23}E_c)\,b^2 + (n_c^{-2}+r_{31}E_a+r_{32}E_b+r_{33}E_c)\,c^2 + 2\,(r_{41}E_a+r_{42}E_b+r_{43}E_c)\,bc + 2\,(r_{51}E_a+r_{52}E_b+r_{53}E_c)\,ac + 2\,(r_{61}E_a+r_{62}E_b+r_{63}E_c)\,ab = 1 \quad \text{(A.1)}$$

Where; E_a, E_b and E_c are the electric field components in the directions of crystallographic axes a, b and c, respectively. So, there are six positions (cases) for the orientation of coordinate system axes (x, y and z) with respect to crystallographic axes. Z-cut (Case1: x//a, y//c, z//b, and case3: x//c, y//a, z//b), Y-cut (case2: x//b, y//c, z//a, case4: x//c, y//b, z//a) and X-cut (case5: x//a, y//b, z//c, case 6: x//b, y//a, z//c). For some cases, to eliminate the cross product terms of IE, the normal plane (normal to the propagation direction) of crystallographic axes must be rotate by an angle 0. Change of refractive index as a function of E_y with E_x=E_z=0, and numerical values are indicated in Tables A.1 and A.2.

Table A.1: Change of refractive index with constant E_y (E_x=E_z=0) and propagation in Z-direction , $A_n=(1/n^2_x - 1/n^2_y)$.
For any hidden cases the values of θ, Δn_x and Δn_y are zeros

Case	Θ	$\Delta n_x/(0.5n^3_x)$	$\Delta n_y/(0.5n^3_y)$
	$LiNbO_3$, $LiTaO_3$		
1	0	$-r_{13}E_y$	$-r_{33}E_y$
2	0	$-r_{23}E_y$	$-r_{33}E_y$
3	$0.5\tan^{-1}(2r_{51}E_y/A_n)$	$A_n\sin^2(\theta)$ $-r_{51}E_y\sin 2\theta$	$-A_n\sin^2(\theta)$ $+r_{51}E_y\sin 2\theta$
4	$0.5\tan^{-1}[2r_{42}E_y/(A_n-r_{22}E_y)]$	$A_n\sin^2(\theta)$ $-r_{42}E_y\sin 2\theta$ $-r_{22}E_y\sin^2(\theta)$	$-A_n\sin^2(\theta)$ $+r_{42}E_y\sin 2\theta$ $+r_{22}E_y\sin^2(\theta)$
5	0	$-r_{12}E_y$	$-r_{22}E_y$
6	$0.5\tan^{-1}(2r_{61}E_y/A_n)$	$A_n\sin^2(\theta)$ $-r_{61}E_y\sin 2\theta$	$-A_n\sin^2(\theta)$ $+r_{61}E_y\sin 2\theta$
	$KNbO_3$, $BaTiO_3$, $BaTiO_5$		
1	0	$-r_{13}E_y$	$-r_{33}E_y$
2	0	$-r_{23}E_y$	$-r_{33}E_y$
3	$0.5\tan^{-1}(2r_{51}E_y/A_n)$	$A_n\sin^2(\theta)$ $-r_{51}E_y\sin 2\theta$	$-A_n\sin^2(\theta)$ $+r_{51}E_y\sin 2\theta$
4	$0.5\tan^{-1}(2r_{42}E_y/A_n)$	$A_n\sin^2(\theta)$ $-r_{42}E_y\sin 2\theta$	$-A_n\sin^2(\theta)$ $+r_{42}E_y\sin 2\theta$

Table A.2 Values of Δn_x and Δn_y with E_y =1 V/ µm (E_x=E_y=0) and case 1 (θ=0)

	$1000\Delta n_x$	$1000\Delta n_y$
$LiNbO_3$	-0.05734	-0.16451
$LiTaO_3$	-.04327	-0.15799
$KNbO_3$	-0.16593	-0.32653
$BaTiO_3$	-0.13298	-0.18402
$BaTiO_5$	-0.11058·	-0.74960

Appendix B: Coupling Coefficient C [13-15]

Coupling coefficient between two slabs directional coupler, slabs A and B, (Fig. B1) is defined as [14] ;

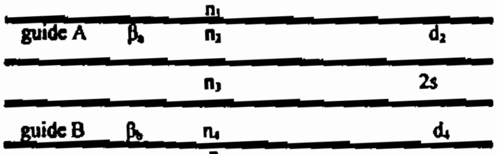

Fig. B.1 Two slabs waveguides directional coupler

$$C^2=(\beta_e-\beta_o)^2/4 - (\beta_a-\beta_b)^2/4 \quad \text{(B.1)}$$

Where, β_e and β_o are even and odd coupled modes, β_a and β_b are the propagation constant for individual two slabs. The difference β_e-β_o is defined by coupled mode theory as [14];

$$\beta_e-\beta_o = [k^4_o k_{ab}k_{ba}+0.25(\beta_a-\beta_b)^2(\beta_a+\beta_b)^2(1-R^2)^2]^{0.5} / [0.5((\beta_a+\beta_b)(1-R^2)] \quad \text{(B.2)}$$

Then,

$$C=[k^4_o k_{ab}k_{ba}]^{0.5} / [(\beta_a+\beta_b)(1-R^2)] \quad \text{(B.3)}$$

Where, R is overlap integral ,

$$R = 0.5 \int_{-\infty}^{\infty} (\psi_a\psi_b + \psi_b\psi_a\, dx = (I_1+I_2+I_3+I_4+I_5) \quad \text{(B.4)}$$

k_{ab} is the coupling coefficient between guides A and B,

$$k_{ab}=(n^2_1-n^2_3)\left[\int_{s+d2}^{\infty}\psi_{x2}\psi_{x4}dx - R\int_{s+d2}^{\infty}\psi^2_{x4}dx\right] + (n^2_2-n^2_3)\left[\int_{s}^{s+d2}\psi_{x2}\psi_{x4}dx - R\int_{s}^{s+d2}\psi^2_{x4}dx\right] = (n^2_1-n^2_3)(I_5-R\,I_9)+(n^2_2-n^2_3)(I_4-R\,I_8) \quad \text{(B.5)}$$

k_{ba} is the coupling coefficient between guides B and A,

$$k_{ba}=(n^2_5-n^2_3)\left[\int_{-\infty}^{-s-d4}\psi_{x4}\psi_{x2}dx - R\int_{-\infty}^{-s-d4}\psi^2_{x2}dx\right] + (n^2_2-n^2_3)\left[\int_{-s-d4}^{-s}\psi_{x4}\psi_{x2}dx - R\int_{-s-d4}^{-s}\psi^2_{x2}dx\right] = (n^2_5 n^2_3)(I_1-R\,I_6)+(n^2_4-n^2_3)(I_2-R\,I_7) \quad \text{(B.6)}$$

By solving eigrn value equation for each individual slab, the values of k_j, β_j, N_j, $\gamma_{j-1,j}$ and $\gamma_{j+1,j}$ (j=1,2) can be determined and the values of I_1---I_9 are,

$$I_1=w.\,e^{-\gamma 32(2s+d4)} / [R_{32}R_{54}(\gamma_{54}+\gamma_{32})]$$

$$I_2=w.e^{-2\gamma 32s}[\,e^{-\gamma 32d4}(m^2_4\gamma_{54}-m^2_5\gamma_{32})/R_{54} + (m^2_3\gamma_{54}-m^2_4\gamma_{34})/R_{34}]/[m^2_4R_{32}(k^2_4+\gamma^2_{32})]$$

$$I_3=w.e^{-s(\gamma 32+\gamma 54)}\,2s/(R_{32}R_{34})$$

$$I_4=w.e^{-2\gamma 34s}[\,e^{-\gamma 34d2}(m^2_2\gamma_{12}-m^2_5\gamma_{34})/R_{12} + (m^2_3\gamma_{12}-m^2_2\gamma_{32})/R_{32}]/[m^2_2R_{34}(k^2_2+\gamma^2_{34})]$$

$$I_5=w.\,e^{-\gamma 34(2s+d2)} / [R_{34}R_{12}(\gamma_{12}+\gamma_{34})]$$

$$I_6=w_1.\,e^{-\gamma 32(4s+2d4)} / (2\gamma_{32}R^2_{32})$$

$$I_7=w_1.\,e^{-\gamma 32(4s+d4)}\sinh(\gamma_{32}d_4)/(\gamma_{32}.\,R^2_{32})$$

$$I_8=w_2.\,e^{-\gamma 34(4s+2d2)} / (2\gamma_{34}R^2_{34})$$

$$I_9=w_2.\,e^{-\gamma 34(4s+d2)}\sinh(\gamma_{34}d_2)/(\gamma_{34}.\,R^2_{34})$$

Where;

$$w=2m_2m_4N_2N_4k_2k_4/(d_{ef2}\,d_{ef4})^{0.5},$$

$$w_1=2m^2_2N^2_2k^2_2/d_{ef2}, \quad w_2=2m^2_4N^2_4k^2_4/d_{ef4}$$

$$R_{12}=(m^4_1k^2_2+m^4_2\gamma^2_{12})^{0.5}, \quad R_{32}=(m^4_3k^2_2+m^4_2\gamma^2_{32})^{0.5},$$

$$R_{34}=(m^4_3k^2_4+m^4_4\gamma^2_{34})^{0.5}$$

$$R_{54}=(m^4_5k^2_4+m^4_4\gamma^2_{54})^{0.5} \text{ and}$$

$$d_{ef}j=d_j+1/(q_{j-1,j}.\gamma_{j-1,j})+1/(q_{j+1,j}.\gamma_{j+1,j}) \quad \text{(B.7)}$$

for TE mode m_j=1, $q_{i-1,j} = q_{j+1,j}$ =1, For TM mode $m_j=n_j$, $q_{j-1,j}=(N_j/n_j)^2+(N_j/n_{j-1})^2-1$, $q_{j+1,j}=(N_j/n_j)^2+(N_j/n_{j+1})^2-1$: j=1,2.

Appendix C: Data of used electrooptic materials [18-26]

For the calculation, we need the whole set of EO material parameters, such as elements of ε_i, r_{ij}, and n_i, we could not find a complete set of these parameters, especially the effect of λ on the last parameters, therefore, we use the following parameters, which we believe, are not too far from the real ones; [27]. Data of EO materials at λ=0.633 µm [18-26] are:

LiNbO3 (n_a=n_b=2.286, n_c=2.200, ε_a=ε_b=43, ε_c=28, r_{12}=-6.8, r_{13}=9.6, r_{22}=6.8, r_{23}=8.6, r_{33}=30.9, r_{42}=32.6, r_{51}=32.6, r_{61}=6.8),

LiTaO3 (n_a=n_b=2.176, n_c=2.180, ε_a=ε_b=42, ε_c=41, r_{12}=-0.2, r_{13}=8.4, r_{22}=0.2, r_{23}=8.4, r_{33}=30.5, r_{42}=20, r_{51}=20, r_{61}=0.2),

BaTiO3 (n_a=n_b=2.410, n_c=2.360, ε_a=ε_b=2300, ε_c=60, r_{13}=19, r_{23}=19, r_{33}=28, r_{42}=820, r_{53}=820),

BaTiO5 (n_a=n_b=2.480, n_c=2.426, ε_a=ε_b=4300, ε_c=168, r_{13}=14.5, r_{23}=14.5, r_{33}=105, r_{42}=1700, r_{53}=1700),

KNbO3 (n_a=2.280, n_b=2.329, n_c=2.169, ε_a=160, ε_b=1000, ε_c=55, r_{13}=28, r_{23}=1.3, r_{33}=64, r_{42}=380, r_{51}=105).

LiNbO3 at λ=1.15 µm (n_a=n_b=2.229, n_c=2.150, ε_a=ε_b=43, ε_c=28, r_{12}=-5.4, r_{13}=9.6, r_{22}=5.4, r_{23}=9.6, r_{33}=30.9, r_{42}=32.6, r_{51}=32.6, r_{61}=5.4).

Note, in this paper we assume quasi-static operations.

To study the effect of wavelength, all the above parameters are assumed independent upon wavelength.

Appendix D:

Dependence Of the Values of $DN=N_2-N_1$ On The Applied Voltages V_1 and V_2 For several electroopic materials

with Case 1, $W_1=W_2=2$ μm, T=4 μm, S=1 μm, , $n_s=1.5$, λ=0.633 μm and E^x_{pq} .

Table D.1: Dependence of values of $DN=N_2-N_1$ on the applied Voltages V_1 and V_2 with case 1, $M_1=M_2$.

$M_1=M_2$	$V_1=0$		$V_1=50$	
	$V_2=0$	$V_2=100$	$V_2=0$ volt	$V_2=100$
$LiNbO_3$	0	0.0022612	-0.0011306	0.0011306
$LiTaO_3$	0	0.0024593	-0.0012295	0.0012298
$KNbO_3$	0	0.0036421	-0.0018210	0.0018210
$BaTiO_3$	0	0.0006413	-0.0003207	0.0003207
$BaTiO_5$	0	0.0021434	-0.0010717	0.0010717

Table D.2: Dependence of values of $DN=N_2-N_1$ on the applied voltages V_1 and V_2 with case 1, M_1=LiNbO.

M_2	V_1 (volt) = 0		V_1 (volt) = 50	
	$V_2=0$	$V_2=100$	$V_2=0$	$V_2=100$
$LiTaO_3$	0.06565	0.06811	0.0645	0.06698
$KNbO_3$	0.01778	0.02142	0.0167	0.02029
$BaTiO_3$	-0.14436	-0.14372	-0.14550	-0.14485
$BaTiO_5$	-0.21231	-0.21017	-0.21344	0.21130

Table D.3: Dependence of values of $DN=N_2-N_1$ on the applied voltages V_1 and V_2 with case 1, $M_1=LiTaO_3$.

M_2	V_1 (volt) = 0		V_1 (volt) = 50	
	$V_2=0$ volt	$V_2=100$	$V_2=0$ volt	$V_2=100$
$LiNbO_3$	-0.06565	-0.06339	-0.06689	-0.06462
$KNbO_3$	-0.04787	-0.04423	-0.04910	-0.04546
$BaTiO_3$	-0.21002	-0.20937	-0.21125	-0.21060
$BaTiO_5$	-0.27796	-0.27582	-0.27919	-0.27705

Table D.4: Dependence of values of $DN=N_2-N_1$ on the applied voltages V_1 and V_2 with case 1, $M_1=KNbO_3$.

M_2	V_1 (volt) = 0		V_1 (volt) = 50	
	$V_2=0$ volt	$V_2=100$	$V_2=0$ volt	$V_2=100$
$LiNbO_3$	-0.01778	-0.01552	-0.01960	-0.01734
$LiTaO_3$	0.04787	0.05033	0.04605	0.04851
$BaTiO_3$	-0.16215	-0.16150	-0.16397	-0.16332
$BaTiO_5$	-0.23009	-0.22795	-0.23191	-0.22977

Table D.5: Dependence of values of $DN=N_2-N_1$ on the applied voltages V_1 and V_2 with case 1, $M_1=BaTiO_3$.

M_2	V_1 (volt) = 0		V_1 (volt) = 50	
	$V_2=0$ volt	$V_2=100$	$V_2=0$ volt	$V_2=100$
$LiNbO_3$	0.14436	0.14663	0.14404	0.14631
$LiTaO_3$	0.21002	0.2124751	0.2096951	0.21215
$KNbO_3$	0.16214	0.1657867	0.1618240	0.16547
$BaTiO_5$	-0.06794	-0.06580	-0.06826	-0.066121

Table D.6: Dependence of values of $DN=N_2-N_1$ on the applied voltages V_1 and V_2 with case 1, $M_1=BaTiO_3$.

M_2	V_1 (volt) = 0		V_1 (volt) = 50	
	$V_2=0$ volt	$V_2=100$	$V_2=0$ volt	$V_2=100$
$LiNbO_3$	0.21231	0.21457	0.21124	0.02135
$LiTaO_3$	0.2779596	0.2804189	0.2768879	0.27935
$KNbO_3$	0.2300885	0.2337306	0.2290168	0.23266
$BaTiO_3$	0.0679438	0.0685852	0.0668721	0.06751