



Answer the following questions

Question 1 32 marks

a) Complete each of the following:

12 marks

- i) The **substitution** used to evaluate the integral $\int \frac{1}{x^2\sqrt{4+x^2}} dx$ isor.....
- ii) The **method** of evaluating the integral $\int \sqrt{8x-x^2} dx$ is
- iii) The **method** of evaluating the integral $\int \frac{e^x}{e^{4x}-1} dx$ is
- iv) If $\int e^{f(x)} \operatorname{sech} x \tanh x dx = e^{f(x)} + c$, then $f(x) =$
- v) $\int \frac{1+\sin^{-1} x}{\sqrt{1-x^2}} dx =$
- vi) $\int \frac{\tan x}{\sqrt{1+\ln^2 \cos x}} dx =$

b) Evaluate each of the following:

12 marks

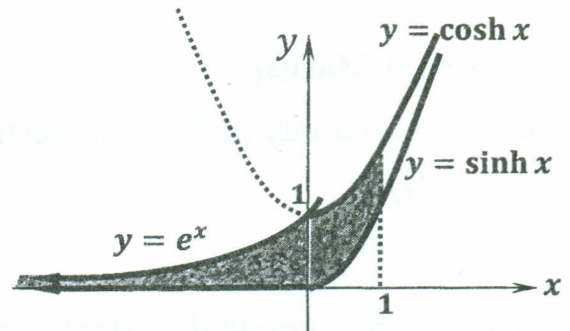
i) $\int \frac{x^2 + 2x}{(1-x^2)^{3/2}} dx$

ii) $\int \ln(x^2 + 1) dx$

c) Find the **volume** of the solid generated

8 marks

by revolving the shaded area about the x - axis.



Question 2 33 marks

a) Evaluate each of the following

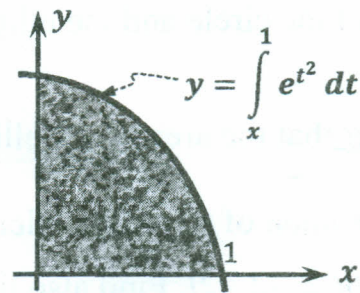
12 marks

i) $\int_{-1}^e f(x) dx$, where $f(x) = \begin{cases} \frac{\ln x^x}{x^2}, & x > 1 \\ x^8 \sin x, & x \leq 1 \end{cases}$

ii) $\lim_{x \rightarrow 0} \left(\frac{\int_0^{x^2} \sinh^{-1} \sqrt{t} dt}{1 - \cos x} \right)$

b) Find the **area** of the shaded region

9 marks



c) The equation $S = 2\pi \int_0^\pi h(x) \sqrt{1 + \cos^2 x} e^{2 \sin x} dx$, represents the **area of the surface** generated by rotating an **arc** passing through the point $(0,2)$.

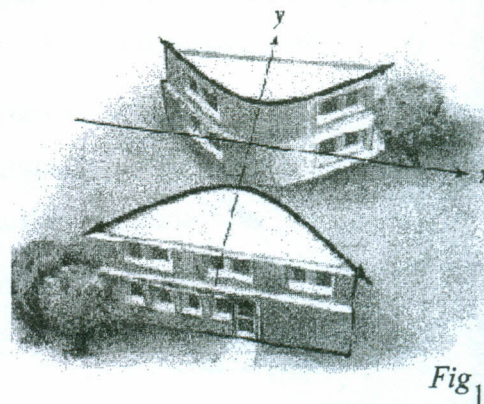
Using this equation, complete each of the following:

12 marks

- i) The equation of the arc is $y =$, $\leq x \leq$
- ii) If the axis of rotation is the y - axis, then $h(x) =$
- iii) If the axis of rotation is the x - axis, then $h(x) =$
- iv) If $h(x) = x + 1$, then the axis of rotation is
- v) The given equation gives value of the length of the arc if we let $h(x) =$

Question 3 [34 Marks]

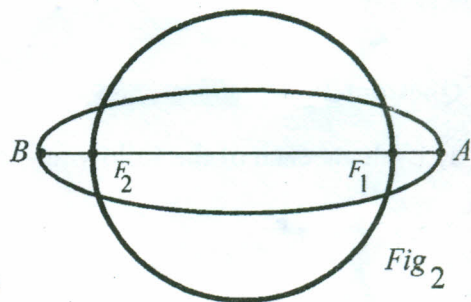
- a) Find the equation of bisectors and the angle between the pair of lines: $x^2 - y^2 = 0$.
- b) Find the equation of parabola whose focus is $(0, 0)$ and its directrix is: $x + y - 3 = 0$. Sketch the parabola and find its main axis.
- c) Identify the conic curve: $x^2 - 2xy + y^2 + 6x + 6y - 9 = 0$.
- d) An architect designs two houses that are shaped and positioned like a part of the branches of hyperbola whose equation is $625y^2 - 400x^2 = 250000$. How far apart are the houses at their closest point?. Find also, the asymptotes of the hyperbola.



Fig₁

Question 4 [31 Marks]

- a) Find the eccentricity of the conic section whose parametric equation is: $x = 2 + 3 \cos t$ and $y = 3 + 2 \sin t$.
- b) In Fig. 2, AB represent the major axis of the ellipse where $A(10, 4)$, $B(2, 4)$ and F_1F_2 represent the diameter of the circle where F_1 and F_2 are the two foci of the ellipse. Find the equations of the circle and the ellipse if: $(\text{area of the circle}) = \frac{3}{2} (\text{area of the ellipse})$



Fig₂

[Remember that the area of the ellipse $\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$ is πab].

- c) Find the equation of the perpendicular bisecting plane to the line segment AB where, $A(2, 2, 2)$, $B(-2, -2, -2)$. Find also the equation of the line AB .
(مطلوب معادلة المستوى العمودى على القطعة المستقيمة من منتصفها)
- d) Find the equation of sphere whose center is $(1, 3, 5)$ and touches the xy -plane. Find the points of intersection of the sphere with z -axis.

Best of luck: Prof. Ibrahim El-Kalla, Dr. Reda Abdou and Dr. Mohammed Soror