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THE RELAXATION TIME MODEL OF TURBULENCE

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الخلاصة :

يقدم هذا البحث تطويرا جديدا للنموذج الرياضى للاضطراب $K - \epsilon$. هذا التعديل يمكن النموذج الرياضى الجديد من التنبأ بطريقة صحيحة بخواص الانسياب النافورى حيث أنه من المعلوم أن النموذج الأصلى يتوقع معدل انتشار لهذا الانسياب يقدر بـ ١١٤ر. بينما التجارب المعملية تقدره بـ ٠٨٦ر. فى هذا البحث تم التركيز على معامل اللزوجة C_μ والذى أثبتت التجارب أنه لا يمكن اعتباره ثابتا إلا فى حالات الانسياب البسيطة . فى النموذج الجديد يتم تغيير قيمة معامل اللزوجة من مقطع إلى آخر وفقا للمتغيرات الأساسية بالانسياب مما يؤدى إلى تقليل إجهاد القص وبالتالى معدل الانتشار النافورى . كما قام النموذج الجديد بالتنبأ بباقى خواص الانسياب الأخرى ووجد أنها متطابقة مع القيم المعملية فى حدود الدقة المسموح بها فى هذا النوع من الانسياب عالى الاضطراب مما يدل على أن النموذج الجديد أقرب إلى الواقع .

ABSTRACT:

This paper presents a new modification to the k-ε model of turbulence to improve the simulation of the turbulent round jet flow. It recommends the use of the "Eddy Relaxation" effect which has the same "Eddy Viscosity" assumption; $-\overline{uv} = \nu_t \ (\partial U/\partial r)$ where $\nu_t = C_\mu (k^2/\varepsilon)$ with C_μ constant in the radial direction; but C_μ adjusted at each downstream station by the local flow parameters. The explanation of the idea is presented. The model is implemented in a finite difference program to solve its equations. Adjustments are made to the time relaxation coefficient β in order to give best agreement between the predicted and the measured spreading rate of the jet half-width. Comparisons for the jet spreading rate, jet half width, and the decay of the jet center-line velocity, with the downstream distance x, are presented for the predicted results by both the original and modified k-ε models as well as the experimental measurements. Mean velocity profiles, turbulent kinetic energy profiles, and shear stress profiles in the self-preserving region ,X/D>70, are compared too. The results indicate that the new model predicts correctly the behavior of the round jet flow in the stagnant surrounding and the agreement with the experimental measurements is within the experimental accuracy.

1- INTRODUCTION :

The turbulent round jet is of a great practical importance and it has peculiarities that introduce difficulties for various computer models of turbulence. Employing the different turbulence models, the velocity field in a two-dimensional plane jet is calculated quite accurately but large errors occur for round jet. Specifically, the spreading rate ($dy_{1/2}/dx$); where ($y_{1/2}$) is the distance from the center-line to the location where the velocity is half the center-line velocity; of the round jet is over-estimated by about 40 %. Experiments show that the rate of spreading, for the half width, is 0.086 for round free jets while the k-ϵ mode [1], for example, predicts a value of 0.114 for the same flow. In the round jets, mean and turbulent velocities decay rapidly in the stream wise direction. Therefore, one might infer that, whenever external conditions produced either by rapid changes or by large imbalances between generative and destructive agencies, available models were liable to give anomalous results.

Here, the simpler mean-flow closure is focused on the k-ϵ model. For constant (unit) density flow, this model determines the Reynolds stress through the isotropic viscosity hypothesis :

$$\overline{u_i u_j} = \frac{2}{3} \delta_{ij} k - 2 \nu_t S_{ij} \qquad \text{........(1)}$$

and in particular

$$- \overline{uv} = \nu_t \frac{\partial U}{\partial y} \qquad \text{..................(2)}$$

where

δ_{ij} is the KRONECKER delta and

$$S_{ij} = \text{mean strain} = \frac{1}{2} \left[\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right]$$

and ν_t is the eddy viscosity which can be related to the turbulent structure of the flow by the Prandtl-Kolmogorov [2,3] relationship as follows:-

$$\nu_t = C_\mu \frac{k^2}{\epsilon} \qquad (3)$$

where C_μ is a constant, k is the turbulent kinetic energy ($\frac{1}{2} u_i^2$)

and ϵ is the dissipation rate of k ($\epsilon = \nu_t \left[\frac{\partial \overline{u_i}}{\partial x_j} \right]^2$)

k and ϵ are determined from the transport equations :-

$$\frac{Dk}{Dt} = T_k + P_k - \epsilon \qquad \text{..................(4)}$$

and $$\frac{D\varepsilon}{Dt} = T_{\varepsilon} + \frac{\varepsilon}{k}\left[C_{\varepsilon 1} P_k - C_{\varepsilon 2}\, \varepsilon \right] \qquad (5)$$

respectively.

The approximate expressions for P_k, T_k and T_{ε} are :-

$$P_k = \text{Production rate of } k \equiv \nu_t \left[\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right] \frac{\partial U_i}{\partial x_j}$$

$$= - \overline{u_i u_j} \frac{\partial U_i}{\partial x_j}$$

$$T_k = \text{Diffusion rate of } k \equiv \frac{\partial}{\partial x_i}\left[\left(\nu_l + \frac{\nu_t}{\sigma_{k,t}} \right) \frac{\partial k}{\partial x_j} \right]$$

$$T_{\varepsilon} = \text{Diffusion rate of } \varepsilon \equiv \frac{\partial}{\partial x_i}\left[\left(\nu_l + \frac{\nu_t}{\sigma_{\varepsilon,t}} \right) \frac{\partial \varepsilon}{\partial x_j} \right]$$

The empirical coefficients widely used in the standard $k-\varepsilon$ model have the following values :-

$C_{\mu} = 0.09$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, $\sigma_{k,t} = 1.0$ and $\sigma_{\varepsilon,t} = 1.3$

with these values, wall boundary layers and the plane jet flow are well predicted.

2- SURVEY OF THE PREVIOUS WORKS :

Equation (5) is the simplest form of transport equation that produces qualitatively correct behavior for ε. In order to achieve quantitatively good predictions, the attention was focused towards the modification of the transport equation of ε. Changing $C_{\varepsilon 1}$ to 1.6 produces the correct rate of spread for round free jets but, in doing that, any notion of generality has to be abandoned. There have been several proposals for extending the width of applicability of equation (5) while retaining some semblance of generality.

Three attempts have been made previously to modify either $C_{\varepsilon 1}$ or $C_{\varepsilon 2}$, all of them make reference to center-line values.

Launder, Morse, Rodi and Spalding [4] suggested the change of $C_{\varepsilon 2}$ and C_{μ} to:

$$C_{\mu} = 0.09 - 0.04\, f$$

and $$C_{\varepsilon 2} = 1.92 - 0.0667\, f \qquad (6)$$

where

$$f = \left| \frac{Y_{1/2}}{2U_o} \left[\left| \frac{d\,U_o}{d\,x} \right| - \frac{d\,U_o}{d\,x} \right] \right|^{0.2}$$

This modification is specially made to fit Rodi's [5] experimental data for round jets and little universality was claimed for it.

McGuirk and Rodi [6] proposed the modification of $C_{\varepsilon 1}$ to:

$$C_{\varepsilon 1} = 1.4 - 5.31 \left[\frac{Y_{1/2}}{U_o} \right] \left[\frac{d\,U_o}{d\,x} \right] \quad \ldots\ldots\ldots(7)$$

The third attempt was from Morse [7], who changed $C_{\varepsilon 1}$ to:

$$C_{\varepsilon 1} = 1.4 - 3.4 \left[\frac{k}{\varepsilon} \frac{\partial\,U}{\partial\,x} \right]^{3}_{x=0} \quad \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

The different modifications produce the same desired effect for self-similar jets but differ slightly in the initial region ($x/d \le 10$) in which Morse's proposal fares best. No convincing physical explanation is provided to justify these modifications.

Pope [8] introduced a term which may be considered as an additional general rate of ε due to the stretching of mean vorticity. Pope argues that the stretching of turbulence vortex lines by the mean flow has a significant influence on the process of scale reduction. On the same scales, the turbulence fluctuations are independent of the mean flow field, hence the vorticity vector has no prefer direction. Since, the vorticity of the large turbulence motion tends to be aligned with the vorticity in the mean flow, the mean straining of turbulent vorticity is strongly corrected with the mean straining of mean vorticity. Thus, in flow regions where the mean vorticity is being stretched; the turbulent vorticity is also being stretched. This leads to greater scale reduction, greater dissipation, less kinetic energy and, consequently, to lower effective viscosity.

In a plane two-dimensional flow, the additional term is supposed to be zero because the mean velocity vector is normal to the plane of the flow. For axisymmetric round jets, however, the mean vortex lines are stretched as the jet enlarges downstream. Therefore, the additional term raises the level of ε and reduces the Reynolds stress and the spreading rate.

The following modification form of the dissipation equation is proposed by Pope:

$$\frac{D\varepsilon}{Dt} = T_\varepsilon + \frac{\varepsilon}{k}\left(C_{\varepsilon 1} P_k - C_{\varepsilon 2}\varepsilon + C_{\varepsilon 3}\varepsilon \chi \right) \quad \ldots\ldots.(9)$$

where $C_{\varepsilon 3}$ is a positive constant, Pope found that a value of 0.79 gives the correct spreading rate for Rodi's jet [3]. χ is the vortex stretching invariant. For axisymmetric flows without swirl χ takes the form:

$$\chi = \frac{1}{4}\left[\frac{k}{\varepsilon}\right]^3 \left[\frac{\partial U}{\partial r} - \frac{\partial V}{\partial x}\right]^2 \frac{V}{r} \quad \ldots\ldots\ldots\ldots(10)$$

where v is the radial turbulence fluctuation component.

Hanjalic and Launder [9] gave another proposal which has certain similarity with Pope's proposal though it has been arrived from a different line of exploration. The concept is that, energy transfer rates across the spectrum are preferentially promoted by irrotational deformation and since energy in transit across the spectrum ends up as energy dissipation, an additional term that promotes higher rates of dissipation for irrotational than rotational strains should be added to equation (5).

That term, for two dimensional thin shear flow, is equal to:

$$-C_{\varepsilon 3}\left(\overline{u^2} - \overline{v^2}\right)\frac{\partial U}{\partial x}\frac{\varepsilon}{k} \quad \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

where $C_{\varepsilon 3}$ is a constant equal to 4.44.

It was proposed:

$$\left(\overline{u^2} - \overline{v^2}\right) = 0.33\ k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Hanjalic and Launder [9] applied the new model; to round jet flow. With the assumption of Equation(12), rate of spread was found to agree with 14 % while the standard $k-\varepsilon$ model gives 40% over prediction.

Yule [10] suggested the use of a modified eddy viscosity coefficient in which C_μ changes with the local flow parameters in the axial direction while it stays constant in the radial one.

3- The present Model:

The standard $k-\varepsilon$ model relates the Reynold's stress tensor

$\overline{u_i u_j}$ to the mean strain S_{ij} using the isotropic viscosity hypothesis Equation (1) as mentioned above. This assumption of the isotropic distribution of ν_t forces the flow to behave as subjected to pure action of strain, which employs the alignment of the principal axes of $\overline{u_i u_j}$ with those of the mean strain S_{ij}. Another weakness is the inability to resolve an isotropy of the normal stresses.

The standard k-ε model employs the assumption that C_μ is constant that produces fairy good results for relatively simple flows in which only one Reynolds stress component is of importance in the momentum equation. However, Rodi [5] showed that C_μ cannot even be considered constant in all simple shear flows. Therefore, the standard k-ε (employing the eddy viscosity concept together with a constant C_μ) lacks the universality required for jet flows because of the inadequate representation in this model of the convective and diffusive transport of $\overline{uv}$.

The above shortcomings indicated the strong need to modify the present hypothesis which presents a direct linkage between the stress field and the mean strain.

To proceed with the problem, one may relate to the situation where the strain field is subjected to a change: S_{ij} can be modified instantaneously while the deviatoric Reynolds stress R_{ij}, being a property of the turbulence vorticity, requires a finite time to change or relax to a new value set by the new mean rate of strain. In other words the large eddies retain their identity with respect to their stress for a time proportional to the turbulence time scale that relates to the large scales, defined as k/ε where the dissipation rate ε represents the rate of energy out of the eddies containing energy. A more explicit expression for the 'lag' effect may be written upon introducing the integral length scale in the time scale expression:

$$\tau' = \beta \, (L/k^{1/2}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

where, L and k represent scales of length and velocity respectively and relate to the large eddies and β the relaxation time coefficient.

The description of the above phenomenon of relaxation is made upon considering the case of a thin shear layer flow where Equation (1) reduces for the shear stress component to the form:

$$\overline{uv} = - C_\mu k^{1/2} L \left(\frac{\partial U}{\partial y} \right) \quad \ldots\ldots\ldots\ldots\ldots(14)$$

The retard may be introduced in the above equation in the following manner:

$$- C'_\mu k^{1/2} L \left(\frac{\partial U}{\partial y} \right) = \overline{uv} + \tau' \frac{\partial \overline{uv}}{\partial t} \quad \ldots\ldots\ldots(15)$$

where C'_μ is a modified eddy viscosity coefficient and τ' is clearly dependent on large eddy energy and dissipation rate.

Equation (15) has quasi-Lagrangian nature, therefore one may use the idea of convection velocity for the eddies such that

$$U_{conv} = 0.5 \left[U_{max} + U_{min} \right] \quad \ldots\ldots\ldots\ldots(16)$$

Therefore, equation (15) becomes,

$$- C'_\mu k^{1/2} L \left(\frac{\partial U}{\partial y} \right) = \overline{uv} + \tau' U_{conv} \frac{\partial \overline{uv}}{\partial x} \quad \ldots\ldots\ldots(17)$$

Substituting from equation (13) into equation (17) yields,

$$- C'_\mu k^{1/2} L \left(\frac{\partial U}{\partial y} \right) = \overline{uv} + \beta \left(L/k^{1/2} \right) U_{conv} \frac{\partial \overline{uv}}{\partial x} \quad \ldots\ldots\ldots(18)$$

Considering the case of a free jet in its downstream self-preserving region, where the evaluation of the flow can be determined directly by the local scales of length and velocity that vary with the downstream distance x and the longitudinal distance y, the above equation can be simplified. Using the fact that the shear stress $\overline{uv}$ can be expressed in the form:

$$\overline{uv} = \overline{uv}_{max}(x) \quad g(\eta)$$

where $\eta = r/L$, the equation (18) becomes,

$$- C'_\mu k^{1/2} L \left(\frac{\partial U}{\partial y} \right) = \overline{uv} + \left[\frac{\beta L}{k^{1/2}} \right] U_{conv} \frac{\overline{uv}}{\overline{uv}_{max}} \frac{\partial \overline{uv}_{max}}{\partial x}$$

$$- \frac{\beta}{k^{1/2}} \; U_{conv} \; \overline{uv}_{max} \; \frac{dL}{dx} \; \eta \; \frac{\partial g}{\partial \eta} \qquad \ldots\ldots.(19)$$

In order to eliminate the effect of the time constant assumption, it seems reasonable to use "typical" values for k and L on R.H.S. e.g. maximum kinetic energy k_{max} and average length scale over the jet-width $\overline{L}$. For a particular value of x, it will be seen that the first two terms on the R.H.S. reduce to the multiple of a constant. The third term contains ,η $(\partial g/\partial \eta)$, that has a radial variation as one crosses the jet from the center-line. The effect of the third term is second order and can be reasonably ignored. Thus, equation (19) can be approximated to:

$$- C'_{\mu} \, k^{1/2} \, L \left(\frac{\partial U}{\partial y} \right) = \overline{uv} + \beta \left[\frac{\overline{L}}{k^{1/2}_{max}} \; U_{conv} \frac{1}{\overline{uv}_{max}} \; \frac{\partial \, \overline{uv}_{max}}{\partial \, x} \right] \overline{uv}$$

or

$$- C'_{\mu} \, k^{1/2} \, L \left(\frac{\partial U}{\partial y} \right) = \overline{uv} \left\{ 1 + \beta \left[\frac{\overline{L}}{k^{1/2}_{max}} \; U_{conv} \frac{1}{\overline{uv}_{max}} \; \frac{\partial \, \overline{uv}_{max}}{\partial \, x} \right] \right\} \, .(20)$$

The above equation indicates that constancy of the bracketed term that multiply β on the R.H.S. of the equation can be achieved when the conditions of self preservation are satisfied where:

$$\overline{uv}_{max} \propto U^2_{max} \propto \frac{1}{x^2} \qquad \text{and} \qquad \overline{L} \propto x$$

In addition

$$U_{conv} \propto \frac{1}{x} \qquad \text{and} \qquad k^{1/2}_{max} \propto U_{max} \propto \frac{1}{x}$$

Therefore;

$$- C'_{\mu} \, k^{1/2} \, L \left(\frac{\partial U}{\partial y} \right) = \overline{uv} \left[1 + \beta \text{ constant} \right] \qquad \ldots\ldots(21)$$

Substituting $\overline{uv}$ from equation (14) into the above equation gives:

$$C'_{\mu} = C_{\mu} \left[1 + \beta \text{ constant} \right] \qquad \ldots\ldots(2$$

where

$$\text{constant} = \left[\frac{\overline{L}}{k^{1/2}_{max}} \; \frac{U_{conv}}{\overline{uv}_{max}} \; \frac{\partial \, \overline{uv}_{max}}{\partial \, x} \right]$$

4. MATHEMATICAL FORMULATION:

Calculations were performed numerically using a finite-difference procedure [11] after implementing the modified model into the code. Because of the parabolic nature of the flow under consideration a forward-marching procedure is used. That means, the properties of the fluid downstream are assumed to be influenced only by the conditions upstream. This procedure is economical because the computer storage required is for two mesh lines only and it is independent of the number of forward steps.

4.1. THE NUMERICAL MESH:

The numerical mesh and control volumes used to calculate any scalar property ϕ are shown in figure (1)

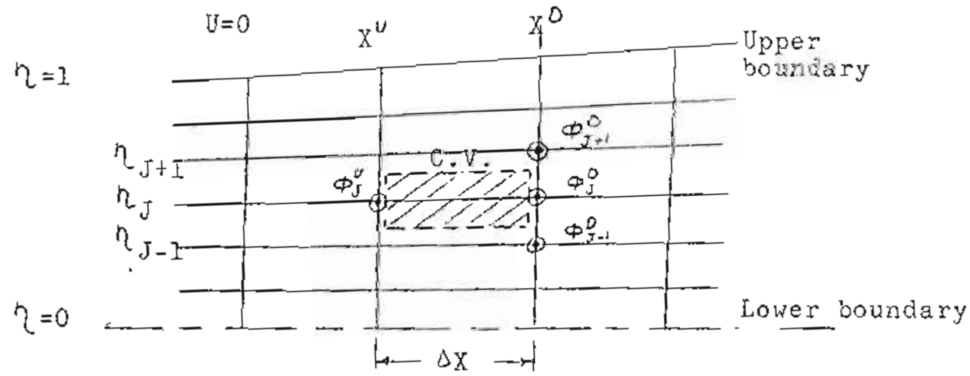

Figure (1) Numerical mach and control volumes

The grid divides the flow width into (n-2) control volumes, where n is the number of nodes. The node positions are at fixed values of η, where $\eta = y/\delta$, but the space between them increases with the increase of the shear layer thickness δ as the flow advances downstream. At all stations, the first and last nodes lie on the lower and upper boundaries respectively. The grid is not uniform but becomes more dense in the regions where the flow variables change rapidly in the direction normal to the axis of symmetry of the jet. Thirty three nodes were used in this investigation.

4.2. THE BOUNDARY AND INITIAL CONDITIONS:

Figure (1) shows that the nodes affecting ϕ^D_J are related to a known upstream scalar value ϕ_J^U and two unknown downstream scalar values ϕ_{J-1}^D and ϕ_{J+1}^D. Therefore, the solution of the algebraic equation system needs the initial and boundary conditions to be known.

The experimental measurements carried out by M. S. Mohamed [12], at the exit plane of a nozzle of diameter D = 0.1 meter and nozzle exit velocity U_J= 62 m/s, are used to prescribe the initial conditions needed in the prediction procedure. The mean velocity component U and the turbulent kinetic energy k were available directly from the measurements. The dissipation rate, on the other hand, could not be directly measured in the experiments. So, it is obtained form the approximate expression relating it to the turbulent kinetic energy k and the mixing length l, described in k-ε model, Jones- Launder [13]

$$\varepsilon = \frac{k^{3/2}}{C_{\mu}^{-3/4}\, l} \qquad \text{..........(23)}$$

The initial mixing length l was divided into two regions according to the initial U mean velocity profile. In the first region, potential flow region where the velocity is uniform and equal to U, the mixing length is assumed to be l=0.09 R (R is the nozzle radius). In the second region, boundary layer region where the velocity decreases rapidly from U to zero, the mixing length is taken l=0.09 δ_i (δ_i is the initial boundary layer thickness).

All over the solution domain, U was kept constant and equal to zero at the upper free stream boundary (at the last node), while k and ε remain as prescribed initially. The lower boundary was taken the axis of symmetry of the jet.

The number of iterations required to give a converging solution was specified by preliminary tests. The tests showed that seven iterations are large enough in order to obtain a stable solution.

5. RESULTS AND DISCUSSION:

The computed and measured behavior for the round jet are compared in figs 2-10 and table (1). Figures (5-10) represent the half of the jet since the flow is symmetric about the center line of the jet.

The solution domain can be divided into three regions, the first one is the near nozzle zone or the potential core region $X/D \leq 7.0$; secondly the transition region $7 < X/D < 70$ and the far downstream region of the jet or the self-preserving region $X/D>70$.

The solution was initially unstable in the potential core region, as shown from the oscillation line in $(dY_{1/2}/dx)$ in fig. (2), therefore a great number of iterations were needed. Then by trial and error, an adjustment of the relaxation time coefficient β to 1.2 was found to predict correctly the spreading rate of the free round jet in a still ambient fluid $(dY_{1/2}/dx=0.086)$ which agrees well with the experimental value [5]. This compared with a value of 0.114 predicted by the standard k-ε model.

Figure (3) shows the change in the dimensionless jet half-width $(Y_{1/2}/D)$ with the downstream distance (X/D). Both models expect the same constant jet half-width in the potential core region while the relaxation time model predicted lower value for $(Y_{1/2}/D)$ than the original model. The modified model responds correctly and agrees with the experimental measurements.

Figure (4) indicates the decay of the jet center line velocity (U_o/U_j) with the downstream distance (X/D). In the potential core region, the jet center-line velocity remains constant and equal to the nozzle velocity U_j. Both models predict correctly the same behavior that agrees well with the experimental data [5]. The relaxation time model computed a lower decay to the jet center-line velocity than the standard k-ε model, in the downstream region of the jet $(X/D>8.0)$, that agrees well with the experimental values.

It may be obvious from the above three figures (2-4) that the original and modified k-ε models predict the same flow field in the potential core region while they have different behavior in the transit and self-preserving regions. The explanation of this behavior is: Near the nozzle the maximum shear stress gradient is approximately equal to zero $(\partial\, \overline{uv}_{max}/\partial x \approx 0.0)$ and in turn $C_\mu^{'} \approx C_\mu$ which means that the correction to C_μ is negligible in this region. This is required because the standard k-ε model is known to predict good results in the two dimensional mixing layer flow which dominates the near nozzle zone. Far downstream of a free jet, in the self preserving region, the new model produces a constant but modified value of C_μ which is also needed to modify the spreading rate of the jet half-width. So, the relaxation time model gives best prediction in the downstream region of the free jet ; a zone in which the standard k-ε model is known to perform badly.

The mean velocity profiles were found to be self similar in the self-preserving region (X/D > 70.), since all of them follow a pattern of a universal curve when normalized with respect to the center-line velocity U_o and jet half-width ($Y_{1/2}$), regardless of the downstream distance (X/D) as shown in figure (5). The self preserving mean velocity profiles predicted by the new model are in excellent agreement with the experimental data [14] as shown in figure (6). The only noticeable difference between the calculated two mean velocity profiles is that the computed one by the modified model approaches the free-stream conditions slowly near the edge of the jet.

The corresponding kinetic energy profiles, shown in fig. (8), also display satisfactory accord in view of the difficulties of obtaining accurate turbulence levels in these high-intensity flows. These curves were also found to be self similar when they were normalized with respect to the square of the jet center-line velocity U_o^2 and the jet half-width ($Y_{1/2}$) as shown in figure (7). The relaxation time model predicts lower kinetic energy k than the original model and both models predict excessive values than the experimental one.

Figure (10) indicates the shear stress profiles across the jet. They were self similar in the self-preserving region when they were normalized with respect to the square of the jet center-line velocity U_o^2 and the jet half-width ($Y_{1/2}$) as shown in figure (9). The relaxation time model calculated lower shear stress values than the original k-ε model. This is expected since the decrease of C_μ will cause a decrease in the effective viscosity ν_t which also decreases the shear stress $\overline{uv}$.

Table (1) shows a comparison between the spreading rate predicted by the different modifications to the k-ε model and the original model as well as the experiments [5]. It is obvious from the table that both the relaxation time model and Pope's model agree well with the experiments. While Hanjalic-Launder's (H-L's) modification improves the solution but it still over estimates the spreading rate.

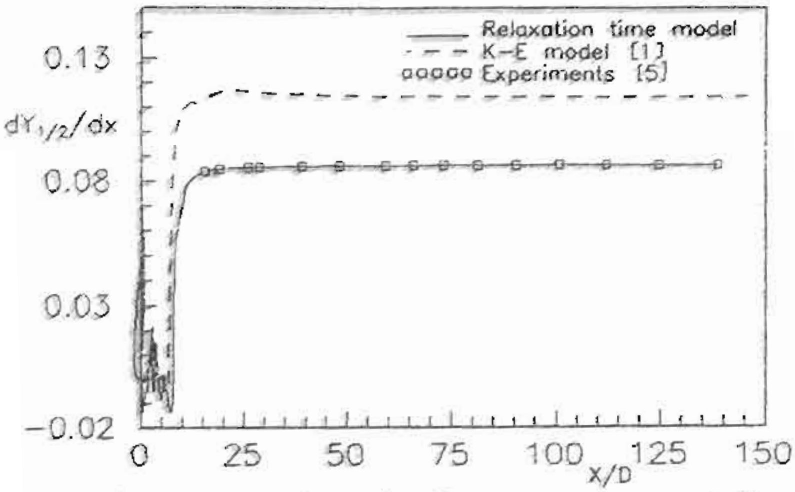

Fig.(2) The spreading rate of the jet half-width with the downstream distance.

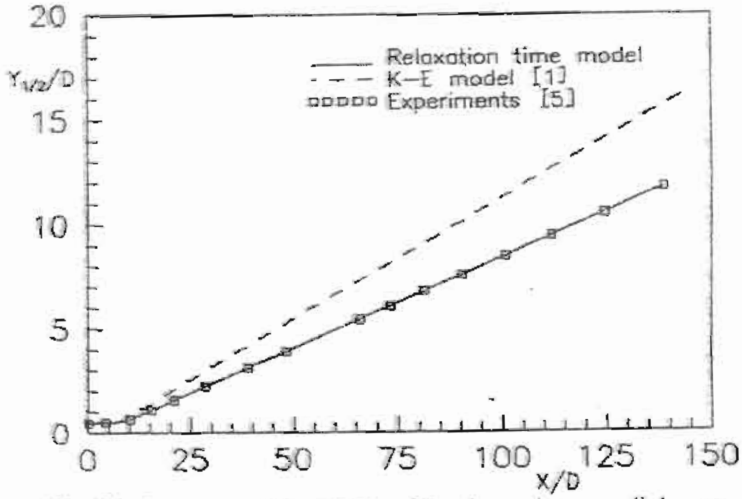

Fig.(3) The jet half-width with downstream distance.

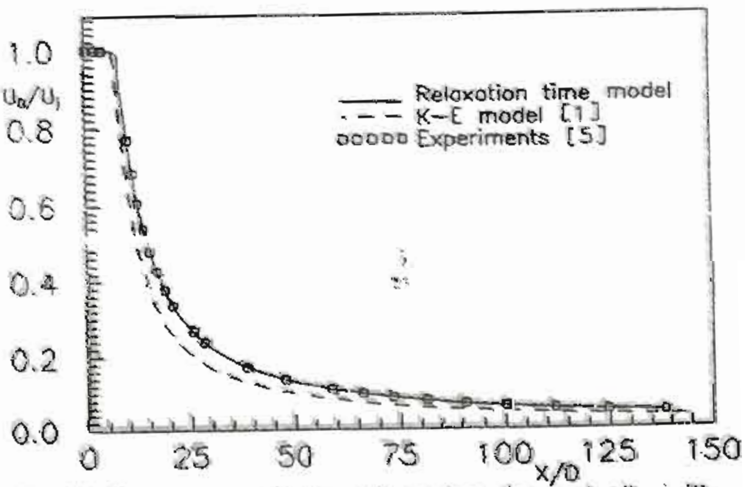

Fig.(4) The decay of the jet center-line velocity with the downstream distance

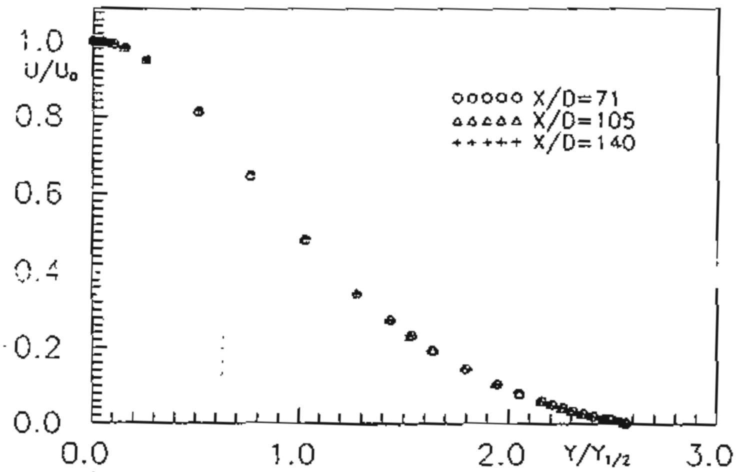

Fig.(5) Self similar dimensionless mean velocity profiles in the self-preserving region.

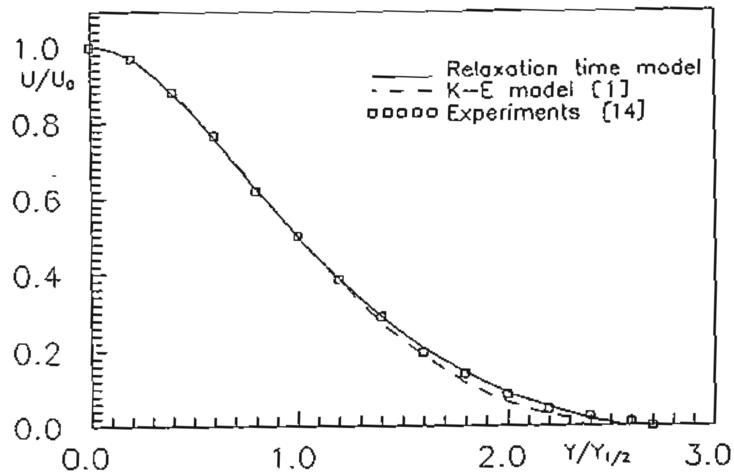

Fig.(6) Comparison between the dimensionless mean velocity profiles.

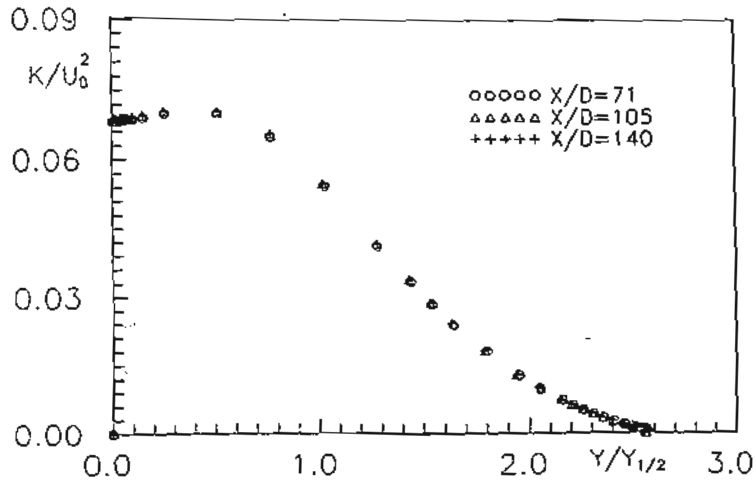

Fig.(7) Self similar dimensionless turbulent kinetic energy profiles in the self-preserving region.

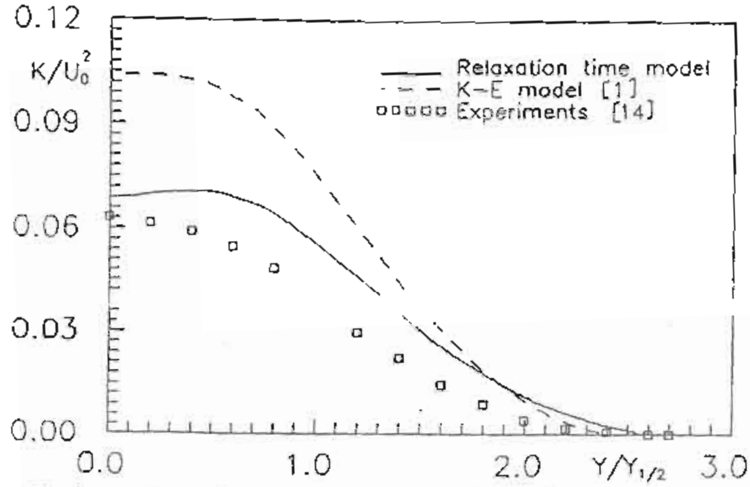

Fig.(8) Comparison between the dimensionless turbulent kinetic energy profiles.

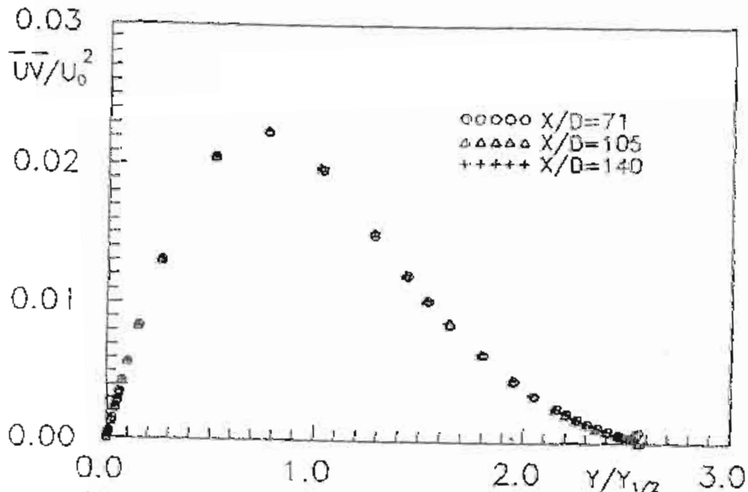

Fig.(9) Self similar dimensionless shear stress profiles in the self-preserving region.

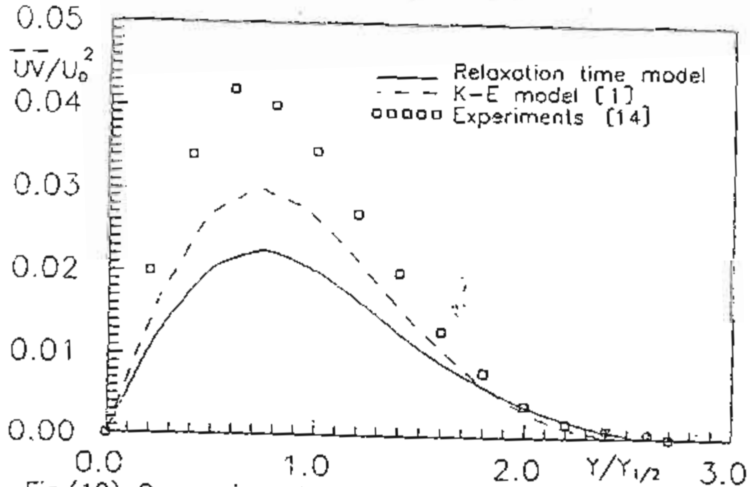

Fig.(10) Comparison between the dimensionless shear stress profiles.

MODEL	Experiments [5]	Relaxation time model	Standard $k-\epsilon$	Pope's Modific	H-L's Modific
Spreading rate	0.086	0.086	0.114	0.086	0.104

Table (1) Spreading rate for round jet in a still ambient.

The eddy Relaxation model uses the Prandtl-Kolmogorov relation between shear and strain but with changing C_μ with the local flow parameters:

$$-\overline{uv} = C_\mu \frac{k^2}{\epsilon} \frac{\partial U}{\partial y} \quad \text{where} \quad \nu_t = C_\mu \frac{k^2}{\epsilon}$$

This produces a variation of C_μ in the axial direction but not in the radial direction, as mentioned above. This is desirable because C_μ is, generally, reasonably constant across free shear layers. It has been shown that the application of the new model in the near nozzle zone, potential core region, $C_\mu = C'_\mu$ which means no change in the predicted velocity profiles by both models because the original model responds correctly in that region. Whereas, in the downstream region, the modified model produces a desired decrease in C_μ and thus a decrease in the jet spreading rate to give excellent agreement with the experiments. The decrease in C_μ causes the reduction of the shear stress $\overline{uv}$ and the turbulent kinetic energy k as mentioned above.

In Pope's modification and Hanjalic - Launder's modification to the original $k-\epsilon$ model, C_μ is constant and equal to 0.09. It recalled that Pope's modification involves an additional term in the ϵ equation. This increases the level ϵ, decreases the kinetic energy k and, consequently leads to lower effective viscosity ν_t that reduces the Reynolds stress and reduces the jet spreading rate, but without directly influencing the value of C_μ. The Hanjalic - Launder's modification similarly results in a decrease in ν_t, however the change would appear to be low as the predicted jet spreading rate is greater than the experimental value.

8-CONCLUSIONS:

The models that use the eddy viscosity concept with constant eddy viscosity coefficient C_μ can predict consistent results for mixing layers and for plane jet as well as wall boundary layers, but the round jet and particularly the round wake requires different value. The new model suggested the employment of a modified eddy viscosity coefficient C'_μ that varies with the local flow parameters in the downstream direction while being kept constant in the radial one. The present model predicts successfully the behavior of the free round jet in stagnant surrounding specially the spreading rate and the decay of the center-line velocity with the downstream distance.

The model shows a realistic behavior even in such details as mean velocity profiles, turbulent kinetic energy profiles and shear stress profiles in the self-preserving region. In general, the agreement lies within the experimental accuracy. The results achieved suggest that this model is close to the truth for the present flow conditions.

7. NOMENCLATURE:

$C_{\varepsilon 1}$, $C_{\varepsilon 2}$	Constants in the k-ε model.
$C_{\varepsilon 3}$	Constant for the additional dissipation term.
C_μ	Eddy viscosity coefficient.
C'_μ	Modified eddy viscosity coefficient.
k	Turbulent kinetic energy.
l	Mixing length.
L	Length scale.
P_k	Production term in the k transport equation.
r	Radial distance.
$r_{1/2}$	Round jet half-width.
T_k, T_ε	Diffusion term in k transport equation and ε transport equation respectively.
U	Mean velocity in the x-direction.
U_j	Nozzle velocity.
U_o	Center-line mean velocity in the x-direction.
u	Longitudinal turbulence fluctuation component.
v	Radial turbulence fluctuation component.
$\overline{uv}$	Reynolds stress.
x, y, z	Longitudinal, radial and tangential dimensional co-ordinates respectively.
$Y_{1/2}$	Half-width for round or plane jets.
ε	Dissipation rate of the turbulent kinetic energy k.
μ	Fluid dynamic viscosity.

ν Fluid kinematic viscosity.

ν_t Turbulent eddy viscosity.

$\sigma_{k,t}$, $\sigma_{\varepsilon,t}$ Turbulent Prandtl number in k and ε transport equations.

β Relaxation time coefficient.

τ Shear stress.

τ' Relaxation time coefficient for eddies.

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