

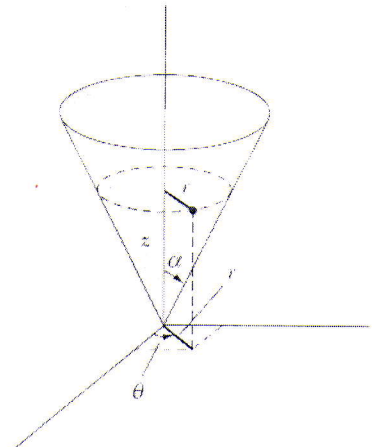


**Answer all questions of the following**

**Question 1**

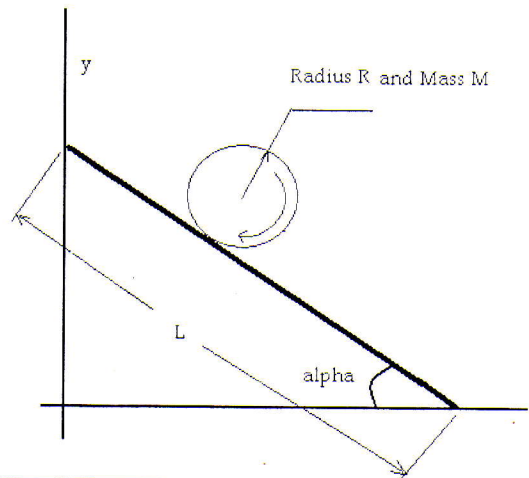
(25 marks)

(A) Suppose that a particle with mass  $m$  is moving in a vertical cone with opening angle  $\alpha$  as shown in the figure. Describe the motion by computing  $\theta$  and  $z$  using Lagrange's equations.



(B) Suppose that a disk with mass  $M$  is rolling on a plane inclined at an angle  $\alpha$  (alpha). As shown, the velocity of the disk is  $\omega = \dot{\theta}$ . Use Euler-Lagrange's equation to describe the motion of the disk with constraint

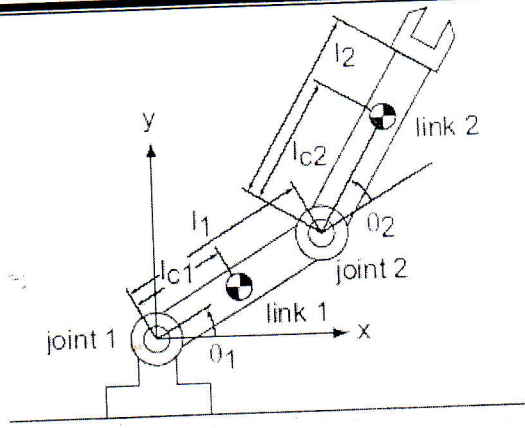
$$g(y, \theta, t) = y - R\theta = 0.$$



**Question 2**

(25 marks)

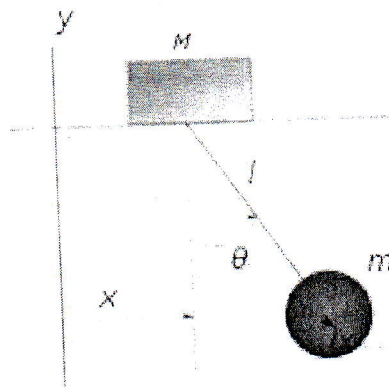
- (A) Derive Lagrange's equations of motion from the principle of least action using elementary calculus. Also, demonstrate the conditions under which energy and momentum are constants of the motion.
- (B) For open link mechanism, write procedure of kinetic and potential energies apply Lagrange's equations of motion and Newton mechanics.



(25 marks)

**Question 3**

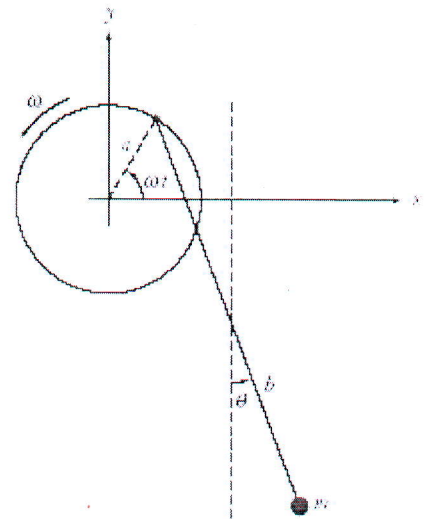
- (A) Suppose that a particle with mass  $m$  is constrained to move on a cylinder with central force  $\vec{F} = -k r \vec{r}$  where  $r$  is the radius of the base of the cylinder. Describe the equations of motion by computing  $z(t)$  and  $\theta(t)$ .
- (B) Consider a pendulum of mass  $m$  and length  $l$ , which is attached to a support with mass  $M$  which can move along a line in the  $x$ -direction. Let  $x$  be the coordinated along the line of the support, and let us denote the position of the pendulum by the angle  $\theta$  from the vertical from the vertical. Describe the kinetic energy, potential Energy, Lagrangian and find the equation of motion



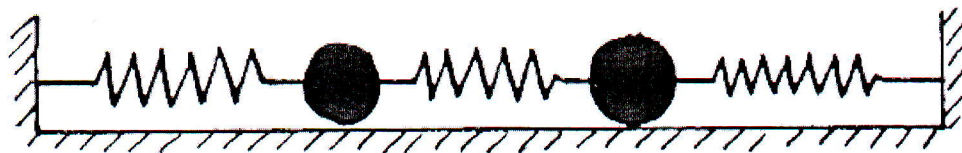
**Question (4)**

(25 marks)

(A) Support of pendulum has a length  $b$  and mass  $m$  are connected with a fixed rotating disk with radius  $a$  and angular velocity  $\omega$  as shown in the figure. Use Euler-Lagrange equations to describe the motion of the pendulum mass.



(B) Two masses are connected with a spring, and each is connected with a spring to a fixed point. Find the equations of motion, and describe the motion qualitatively. Solve for the possible angular frequencies in the case when the masses are equal and the spring constants are equal. There is no friction.



This exam measures the following ILOs							
Question Number	Q1-a	Q2-b	Q3-b	Q4-a	Q1-b	Q3-a	Q4-b
					Q2-a		
Knowledge & understanding skills					Intellectual Skills		Professional Skills

*With my best wishes*

*Dr. Ramzy M. Abumandour*