Menoufiya University
Faculty of Engineering,
Shebin El-Kom
Dep.:Civil Engineering Department
Semester: Second
Academic year:2019-2020

Postgraduate: Master
Subject: : Advanced Fluid Mechanics
Code No.: CVE 640
Date:17/08/2020
Time Allowed: 3.0 hrs.
Total marks:100

Solve the following questions, Any Missing Data Can Be Reasonably Assumed.
Illustrate your answer with neat sketches. Answers should be organized, concise and readable.

Question (1) ---------- **(15 Marks)**

a) Examine the continuity of the following cases as flow:

i) $u = -ax + b$, $v = ay + c$

ii) $u = ax + b$, $v = ay + c$

iii) $u = ax$, $v = ay$, $\omega = -2az$

iv) $u = \frac{x}{x^2+y^2}$, $v = -\frac{y}{x^2+y^2}$

For the case of continuous flow, find the stream-function ψ **(8 Marks)**

b) Examine the condition of irrotationality in the four cases given in (a). For the case of irrotational flow, find the velocity-potential φ and hence sketch the flow- net. **(7 Marks)**

Question (2) ---------- **(15 Marks)**

a) Find the velocity components of the flow whose velocity-potential φ and stream-function ψ are given by:

$$\varphi = U[\, x + \frac{a^2 x}{x^2+y^2}\,] \quad , \quad \psi = [\, y - \frac{a^2 y}{x^2+y^2}\,]$$

Where a and U are given constants. Find the velocity at large distance from 0, the stagnation points and discuss the motion in general. Sketch the flow net. **(8 Marks)**

b) Find the same requirements as in equation (a) for the flow:

$$\varphi = \frac{x}{x^2+y^2} \quad , \quad \psi = -\frac{y}{x^2+y^2}$$ **(7 Marks)**

Question (3) ---------- **(15 Marks)**

a) A flow is given by the following stream functions :

i) $\psi = x^2 - y^2$, ii) $\psi = xy$, iii) $\psi = \log(x^2 + y^2)$

If the pressure distribution is given by :

a) $P = P_0 - 2(x^2 + y^2)$

b) $P = P_0 - \frac{1}{2}(x^2 + y^2) + \frac{1}{2}(x + y)^2$

c) $P = P_0 - \frac{1}{2}\frac{1}{(x2 + y2)} + xy$

Respectively, find the external forces that produced the corresponding motion. **(8Marks)**

b) A clear jet rises vertically from a circular orifice 1 inch in diameter. Neglecting loss by friction at the orifice and in the air, what is the diameter half-way the summit. Use Bernoulli's equation and assume uniform velocity all over any cross-section of the jet. **(7 Marks)**

Question (4) ---------- **(24 Marks)**

a) **Prove that :**

$\cosh i\Theta = \cos \Theta$, $\sinh i\Theta = i \sin\Theta$

$\cosh \Theta = \cos i\Theta$, $I \sinh \Theta = \sin i\Theta$

(8 Marks)

b) **If $z = x + iy$, $z' = x' + iy'$, prove that $z = z' e^{i\alpha}$ turns the axes of reference (x,y) through the angle α, and $z' = z e^{-i\alpha}$ turns the axis (x',y') through the angle α in the opposite sense.**

(8 Marks)

c) **Separate the real and imaginary parts of :**

Z^3 , $\frac{1}{z}$, cos z , sin z , cosh z and sinh z **(8 Marks)**

Question (5) ---------- **(16 Marks)**

a) **Show that the velocity potentials :**

i) $\varphi = \frac{Ux}{x^2+y^2}$, ii) $\varphi = m \tan^{-1} \frac{y}{x}$

give a possible motion, and determine the complex-potential in each case. **(8 Marks)**

b) **If the velocity components are given by :**

i) $u = 1 - \frac{y}{x^2+y^2}$, $v = \frac{x}{x^2+y^2}$

ii) $u = \frac{-x^2+y^2}{x^2+y^2}$, $v = - \frac{2xy}{(x^2+y^2)^2}$

prove that these components represent a possible 2- dimensional irrotational motion, and find the corresponding complex-potential. **(8 Marks)**

Question (6) ---------- **(15 Marks)**

a) **show that the motion whose stream-function is :**

i) $\psi = - m \log (x^2 + y^2)$, ii) $\psi = k \tan^{-1} (\frac{y}{x})$

is irrotational, and find the complex-potential in each case. **(7 Marks)**

b) **If** (a) $\omega = - Uz - m \log z$

(b) $\omega = - m \log (z^2 - a^2)$

(c) $\omega = - Uz - m \log (z^2 - a^2)$

Determine the velocity at large distances from the origin and the stagnation points. Sketch the flow in each case. **(8 Marks)**

With My Best Wishes

Prof. Mohamed Sobeih

This exam measures the following ILOs								
	Q1	Q3	Q4	Q5	Q2	Q4	Q5	Q6
	MK1	MK3	MK2	MK3	MP1	MP4	MP4	MP1
	Knowledge &Understanding Skills				Professional Skills			