CONSTANT WALL TEMPERATURE HEAT TRANSFER IN DRAG REDUCING ADDITIVE FLUID FLOWS

RABIE, L.H., TOLBA, M.A.*, ARAID F.F., and AWAD M.H.
Mechanical Engineering Dept., Faculty of Engineering, Mansoura University,
Mansoura, EGYPT.

ABSTRACT:

This work presents an experimental and theoretical study for heat transfer in a fully developed pipe flow of drag reducing additive fluid. The study is carried out at constant wall temperature heat transfer condition. Different concentrations of polymer additives; 10, 20 and 50 wppm; are used throughout the study. The results are compared with those of pure water flow as a Newtonian. Experimental results show derastic reductions in heat transfer as well as the frictional drag associates the addition of polymer additives to water flow.

Heat transfer predictions were carried out by an analytical model developed by solving the momentum and energy differential equation for the case of constant wall temperature condition. The proposed model uses the modified van Driest eddy diffusivity model in addition to the assumption of a constant value turbulent Prandtl number ($\sigma_t = 0.85$). The predicted results agree very well with the experimental values.

NOMENCLATURE:

A^+	Viscous damping coefficient
A_n^+	Viscous damping coefficient in Newtonian flow
ΔB	Upward shift in log region of the streamwise velocity profile
C_p	Specific heat at constant pressure
f	Friction factor $f = 2\tau_w/\rho U_b^2$
G	Function given by equ.(9)
H	Function given by equ.(10)
k	Von Karman's constant $k = 0.4$
k_c	Thermal conductivity
$N\bar{u}$	Nusselt number
q	Radial heat flux
q_w	Radial heat flux at wall
q^+	Dimensionless radial heat flux $q^+ = q/q_w$
r	Radial distance
R	Pipe radius
R^+	Dimensionless pipe radius $R^+ = R\, u_s/\nu$
Re	Reynolds number, $Re = 2\, U_b\, R/\nu$
St	Stanton number, $St = q_w / \rho C_p\, U_b\, (T_w - T_b)$
St_o	Stanton number based on centreline temperature, $St_o = q_w/\rho C_p\, U_b (T_w - T_o)$
T	Temperature
T^+	Dimensionless temperature, $T^+ = (T_w - T)\, C_p\, \tau_w/q_w\, u_s$
T_b^+	Dimensionless mean bulk temperature
T_o^+	Dimensionless centreline temperature
u	Stream wise velocity
u_s	Shear velocity, $u_s = \sqrt{\tau/\rho}$
u^+	Dimensionless streamwise velocity $u^+ = u/u_s$
U_b^+	Dimensionless mean bulk velocity

* A Ph.D. Student.

u_o^+ Dimensionless centreline velocity
y^+ Dimensionless distance from the wall $y^+ = y\, u_s$
β Constant of equ. (14)
β_H Constant of equ. (12)
ε Eddy diffusivity of momentum
ε_h Eddy diffusivity of heat
ν Kinematic viscosity
ρ Density
σ Prandtle number, $\sigma = \rho C_p / k_c$
σ_t Turbulent prandtl number $\sigma_t = \varepsilon / \varepsilon_h$
τ Radial shear stress
τ_w Wall shear stress
τ^+ Dimensionless radial shear stress, $\tau^+ = \tau / \tau_w$
$\phi = q^+ / \tau^+$

1. INTRODUCTION

Most of heat transfer studies, so for, have been carried out wit. Newtonian fluids, and most often the fluid has been water or air. In fac. this represent the most immediate intrest in every day life as well as industrial processes. However, with the increase of our technology sophistication, a great need for the study of non Newtonian fluids increases and becomes essential. One of the non Newonian fluid types is that of drag reducing additives or the viscoelastic fluids. They have received widespread attention due to the tremendous reduction of the frictional drag that associates their turbulent flow compared with the Newtonian one. A few parts per million of some polymers added to turbulent water flow can reduce the frictional drag by almost 80% [1,2]. In fact, many industrial processes involve fluid flows inherently have drag reducing charactristics such as food processing, paper's production and manufacturing of chemicals. Naturally, primary attention has been given to study the momentum transfer which determines the drag reduction and the changes in the flow structure which associate drag reducing fluid flows compared with those of Newtonian one [1-3]. later on, the interest in heat and mass transfer in these flows increase, and a number of studies is now available. The experimental work of Gupta et al [4], Metzner and Friend [5], Dehrule and Sabersky [6], Smith et al [7] Mc Nally [8], and Kwack et al. [9] show large reduction in heat transfer similar to that of frictional drag. Most of these studies were carried out at constant wall flux condition. The results even show considerable scatter due to shear and thermal degradation. Some analytical and numerical studies have been made to predict heat transfer in drag reducing fluids [11 - 15]. Various assumptions and approximations have been introduced in the different models to solve the boundary layer energy equation. Ghajar and Tiederman [11] uses the eddy diffusivity of Cess[16] together with the experimental data of friction to predict heat transfer coefficient, Dimant and Poreh [12] uses a modefied van Driest [17] eddy diffusivity model to integrate the energy equation numerically. Kale [13] uses the eddy diffusivity of Deissler to formulate an analytical expression for Nu. Smith et al. [14] use four different models for eddy diffusivity to predict heat transfer in drag reducing flows. Their results show that eddy diffusivity models of Mizushina et al. [18] and van Driest [17] have better performance compared with the other considered models.

Yoon and Ghajar [15] in a recent work used the eddy diffusivity due to Cess [16] for momentum and three different models for heat. They recommended Reynolds analogy for dilute solutions. Mizushina and Usui's [19] model at minimum heat transfer condition for the case of concentrated polymer solutions. Each of the proposed models correlates satisfactorily some sets of data and fails to agree with other sets. Even though, the validity and limitations of these models should be evaluated further with more reliable experimental data which covers both constant wall temperature and constant heat flux conditions, wide range of polymer concentration different, flow conditions and different polymer types. Unfortunately this type of data is not available as discussed above. Efforts should be made to generate these data.

In this work an experimental and analytical study are presented for the heat transfer in fully developed pipe flow at constant wall temperature condition. Experiments are carried out with dilute solutions of polyacrylamide at different flow conditions and polymer concentration. An analytical expression for dimensionless heat transfer coefficient (Stanton number) using the eddy diffusivity model of van Driest is given.

2. THEORETICAL BACKGROUND

The analysis presented here is similar to that developed by Rabie [20] for heat transfer in non Newtonian drag reducing fluid flows. In a fully developed pipe flow of an incompressible fluid, one can write the momentum and energy equations as:

$$-\frac{1}{\rho}\frac{dP}{dx} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\,(\nu + \varepsilon)\,\frac{\partial u}{\partial r}\right) \qquad \text{.....(1)}$$

and

$$-u\,\frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left(\frac{k_c}{\rho C_p} + \varepsilon_h\right)\frac{\partial T}{\partial y}\right) \qquad \text{.....(2)}$$

The radial heat and momentum flux can also be written in the form

$$\frac{dT^+}{du^+} = \frac{q^+}{\tau^+}\,(1 + \varepsilon/\nu)\,/\left(\frac{1}{\sigma} + \frac{\varepsilon_h}{\nu}\right) \qquad \text{.....(3)}$$

Most of previous heat transfer models assume that $q^+/\tau^+ = 1$. With this assumption, there is no difference between constant heat flux and constant wall temperature modes of heat transfer. In order to fined the exact relation, the energy and momentum equations are integrated to give the following for the case of constant wall temperature condition.

$$\frac{q^+}{\tau^+} = \frac{\int_0^{y_1^+} (R^+ - y^+)\, u^+ T^+\, dy^+}{U_b^+\; T_b^+ \int_0^{y_1^+} (R^+ - y)\, dy^+} \qquad \text{.....(4)}$$

For flow and heat transfer predictions, the differential equations of momentum and energy (equations 1 and 2) are solved by assuming expressions for eddy diffusivities of momentum and heat (ε & ε_h) respectively. It is very common to relate both heat and momentum eddy diffusivities through the turbulent Prandtle number σ_t ($\sigma_t = \varepsilon / \varepsilon_h$). Hence, only one expression for eddy diffusivity is needed for both momentum and heat. In this work the eddy diffusivity of van Driest is used. It is recommended by many investigators [12 & 18] for flow and heat transfer predictions in drag reducing fluid flows as well as Newtonian ones. It is given as [17]

$$\frac{\varepsilon}{\nu} = (k\, y^+)^2 \left(1 - \exp\,(-\, y^+/A^+)^2\right) \frac{du^+}{dy^+} \qquad \ldots\ldots(5)$$

and

$$\frac{du^+}{dy^+} = \frac{2\tau^+}{1 + \left[1 + 4\,(ky^+)^2\,(1 - \exp\,(-y^+ / A^+)^2\, \tau^+\right]^{\frac{1}{2}}} \qquad \ldots\ldots(6)$$

This model contains two parameters k and A^+. The parameter k is a constant and has a value of 0.4 for both Newtonian and drag reducing fluid flows. A^+ is the viscous damping coefficient which has a value of $\simeq 26$ for Newtonian fluid flows. Its value increases in drag reducing additive flows and depends mainly upon the level of drag reduction. A^+ is related to the upward shift in velocity distribution ΔB as [12].

$$A^+ = A^+_n \quad \exp\,((\,\Delta B + 39.6)/(24.6) - 4.0) \qquad \ldots\ldots(7)$$

where A^+_n is the Newtonian fluid flow value which equals to 26.

Introducing the turbulent Prandtl number and integrating equation (3) from the wall to the centreline gives

$$S_{t_o} = \frac{\sqrt{f/2}}{\sigma_t\, G + (\,\sigma - \sigma_t)H} \qquad \ldots\ldots(8)$$

where;

$$G = \int_0^{R^+} \phi \quad (du^+/dy^+)\, dy^+ \qquad \ldots\ldots(9)$$

$$H = \int_0^{R^+} \phi \quad (du^+/dy^+)/(1 + \frac{\sigma}{\sigma_t}\,\frac{\varepsilon}{\nu})dy^+ \qquad \ldots\ldots(10)$$

and $\qquad \phi = q^+/\tau^+$ which is given by relation (4)

The two integrals G and H were carried out numerically using the eddy diffusivity model of van Driest. The results can be approximated as

$$G \simeq 0.925 \quad \sigma_t\, U^+_o \quad ;\text{and} \qquad \ldots(11)$$

$$H \simeq 8.75 \;\; \sigma^{-0.25} + 0.685\,(\,\Delta B)\;\; \sigma^{-0.22} \qquad \ldots(12)$$

For pipe flow, Stanton number is normally based on bulk temperature T_b. Kader and Yalgom [10] suggested an expression for the relationship beteen St and St_o, based on the assumption that the temperature distribution in turbulent thermal layer is a logarithmic one, as

$$\frac{\sqrt{f/2}}{St} = \frac{\sqrt{f/2}}{St_o} - \beta_H$$

where; $\beta_H = 1.5\ (\frac{\sigma_t}{k})$

In a similar mannr , assuming that the streamwise velocity profile in drag reducing fluid is in the form,

$$u^+ = \frac{1}{k}\ln y^+ + B + \Delta B + \frac{P}{k}(1 - \cos\ \pi y^+/R^+) \qquad \ldots\ldots(13$$

the following relationship between bulk (average) velocity U_b^+ and the centreline one U_o^+ is derived as

$$U_b^+ = U_o^+ - \beta \qquad \ldots\ldots(14$$

where $\beta = \frac{1.5}{k} + \frac{P}{k}(1 - \frac{4}{\pi^2})$

Hence, the dimensionless heat transfer coefficient (Stanton number) can be given as:

$$St = \frac{\sqrt{f/2}}{0.925\ \sigma_t\ (\sqrt{2/f} + \beta) + (\sigma - \sigma_t)\ H - \beta_H} \qquad \ldots\ldots(15$$

For fully developed thermal and hydrodynamic layers, k, σ_t and P are assumed to be 0.4, 0.85 and 0.2 respectively. As a result, β and β_H are found to be 4.07 and 3.375 respectively. Hence equation (15) can be written as:

$$St = \frac{\sqrt{f/2}}{0.786\sqrt{2/f} + (\sigma - 0.85)\ H - 0.175} \qquad \ldots\ldots(16$$

This expression gives the dimensionless heat transfer coefficient with the knowldage of the friction factor "f" and Prandtl number " σ ".

3. EXPERIMENTAL TECHNIQUE

The object of this work is to study heat transfer at constant wall temperature condition in drag reducing additives in comparison with Newtonian (water) flows. Both heat transfer coefficient and friction factor are to be measured. An experimental rig is designed and constructed for that purpose. It is a gravitational open flow system. A schematic diagram of the system is shown in figure (1). It mainly consists of two flow lines: water flow and concentrated polymer solution lines. The two flows supply a mixing chamber where both water and concentrated polymer solutions are mixed teogether to the required concentration. The dilute solution, then flows to the test section and finally to the drain through a centrifugal pump to avoide mechanical degradation of polymer additives before the test section.

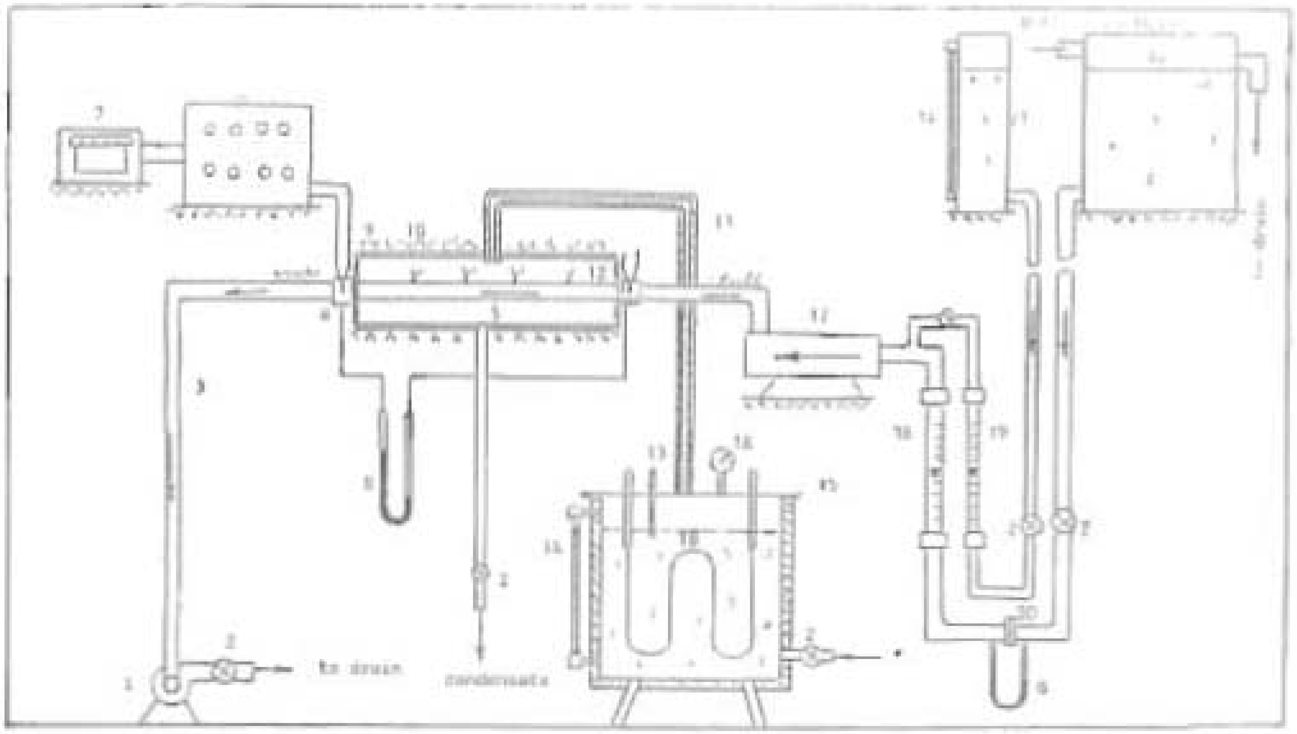

Fig. (1) LAYOUT OF EXPERIMENTAL TEST LOOP.

1- Water pump, 2- Control values, 3- Outlet pipe, 4- Teflon conections, 5- Test tube, 6- Switch board, 7- Multimeter, 8- Differential manometer, 9- Steam drum, 10- Insulation of glass wool, 11- Saturated steam pipe line, 12- Copper-constantan thermocouples, 13- Thermometer, 14- Water level indicator, 15- Electric boiler, 16- Pressure gauge, 17- Mixing chamber, 18- Water flow meter, 19- Polymer solution flow meter, 20- Orifice meter, 21- Polymer soluting tank, 22- Constant head water tank.

As shown in figure (1), water is supplied from a constant head overflow tank of capacity 0.6 m^3. Another one of capacity 0.05 m^3 is used for concentrated polymer solutions. Both tanks are placed at a level 5.5m higher than that of the test section and 7.0 m higher than the discharge level. This ensures reasonable flow rate under the gravity action. However, the pump is sometimes used for higher flow rates. The test section is a 16 mm diameter copper tube of 100 cm length. It is enclosed by 40 mm diameter shell of 100 cm length. The shell is supplied with saturated steam at 100^0 to heat the flow in the test section. The use of saturated steam ensures constant wall temperature condition. The condensate is collected and the rate of condensation is measured. An electric steam generator is used to generate the steam required. To avoid axial flow of heat, the test section is enclosed between two tube sections made of insulating material (Teflon).

Water and concentrated polymer solution flow rates are measured using caliberated orifice and float meters. Control valves are used to control the rate of water and concentrated solution flows to the required flow rate and average concentration of the dilute solution. The test section is provided with two pressure taps 95 cm appart to measure the pressure drop using a U tube manometer. The manometer's liquid used is CCl_4 of 1.593 specific gravity. The inlet and outlet flow temperatures were measured by two cupper - constentan thermocouples fixed at the inlet and outlet of the test section. The test section is also provided with four cupper - constentan thermocouples to measure the average wall temperature. The temperature is measured relative to the ice point ($0\ ^0C$) by electric potential developed across the thermocouples terminales using a digital voltmeter with 0.01 mv resolution. The heat transfer rate is then calculated from the flow rate and the temperature difference between inlet and outlet as;

$$Q = m\ cp\ (t_{b_o} - t_{b_i})$$

Heat transfer coefficient, friction factor and Reynolds number are calculated at mean bulk temperature ($t_b = (t_{b_o} + t_{b_i})\ /2$).

4. RESULTS

It has been demonstrated that the addition of minute quantities of high molecular weight polymers to turbulent flows drastically reduce both momentum and heat transfer [1]. This work presents a study of heat transfer at constant wall temperature condition in a pipe flow of drag reducing additive fluids. The results are shown in figures (2 & 3), where the friction and heat transfer coefficients of water and dilute polyacrylamide solutions are plotted as function of Reynolds number. Throughout this work three polymer concentrations are used 10, 20 and 50 wppm respectively. Experiments were carried out with a fluid temperature of 22^0C at inlet and outlet temperature range of 27.5^0C to 38.5^0C in pure water flow. This corresponds to an average Prandtle number of 6.2 to 5.4 . In dilute solution flows, outlet temperature is in the range of 23.5 to 28 giving an average Prandtle number values of 6.55 to 6.0 .

Figure (2) shows the friction factor "f" as function of Re. Water flow results show slightly higher values of friction factor compared with literatur's data represented by Blasius formula;

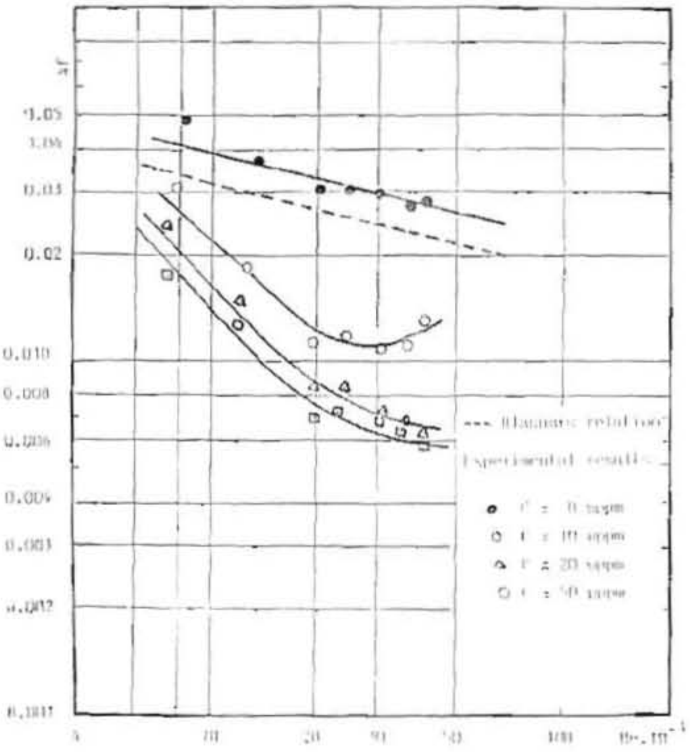

Fig.(2): Relation between friction factor and Reynolds number at different polymer concentration.

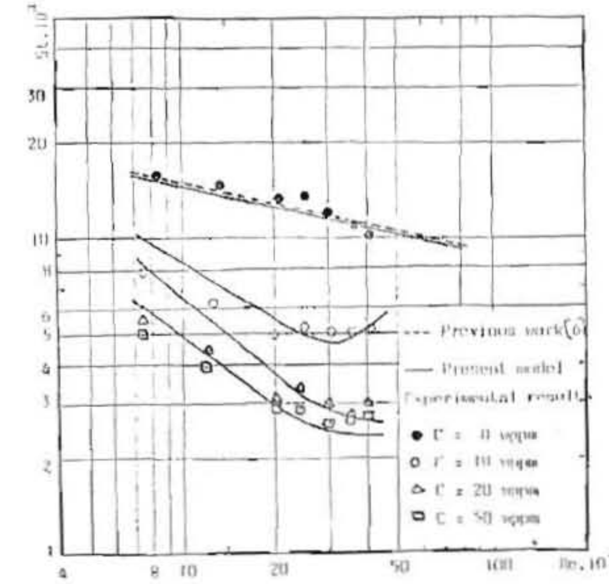

Fig.( ): Relation between Stanton number and Reynolds number at different polymer concentration.

$$f = 0.0791 \ Re^{-0.25}$$

This may be due to the fact that the pipe is not smooth enough. However water flow data has been given for reference purpose only. In dilute polymer solution of 10 wppm, experimental results show reduced values of the friction factor compared with those of water flow data. The reduction in friction factor increases with Re up to 25,000 and detriorates at higher values of Re. This feature is shown previously by Debrule and Sabersky's data 6 but at higher values of Re ($Re \approx 150,000$). This detrioration is thought to be due to shear degradation of polymer molecules. The remarkable low value of Re at which shear degradation occurs is strange and may be due to chemical degradition of the polymer because of long storage (about 5 years). Long storage period acually reduce the average molecular weight of the polymer and reduces its effectiveness as a drag reducer. At $Re = 8000$, 10 wppm concentration gives 35% reduction in friction. This increases to 63% at Re 30,000. For dilute solutions of 20 and 50 wppm concentrations, the results give slightly different picture within the range of Reynolds number of the experiments. The reduction in friction factor increases with Reynolds number with a relaxing effect at Re 30,000 - 40,000, giving a maximum reduction of 75% . and 78% for 20 and 50 wppm respectively. This is in agreement with literatures data [1-5] .

Heat transfer results are shown in figure (3) where experimental values of dimensionless heat transfer coefficient (Stanton number St) are compared with those predicted by the present model. In general, heat transfer results are much similar to those of friction factor for dilute polymer solutions. Although water flow results are shown for the reference purpose only, they show very good agreement with available heat transfer data represented by

$$Nu = 0.021 \ Re^{0.8} \quad \sigma^{0.43} \left(\sigma / \sigma_w \right)^{0.25}$$

as shown in figure (3). The results also show excellent agreement with predicted values. Dilute polyacrylamide solution results exhibit substantial reduction in heat transfer coefficient. For 10 wppm polyacrylamide concentration, the reduction in heat transfer increases with increase in Reynolds number up to $Re \approx 30,000$ and then decreases with further increase in Re This behaviour is similar to that of frictional resistance. At $Re \approx$ ..000, the measured value of heat transfer reductions about 40% compared with a predicted value of 30%. This increases to 60% at $Re = 30,000$ and then decreases to 45% at $Re \approx 50,000$. For 20 and 50 wppm concentration, the reduction in heat transfer associates the addition of polyacrylamide to the flow increases with Reynolds number, e.g. 60% reduction is found at Re 8,000 increases to about 80% at $Re \approx 40,000$. In general, these results show excellent agreement with predicted values at high Reynolds number. But at low Reynolds number, predicted heat transfer results are about 20% higher than those experimentally determined. This difference may be attributed to some experimental errors in measuring the temperature of drag reducing flows as that found with Pitot tube and hot wire (film) anemometer measurements [2] .

5. DISCUSSION

The above mentioned results shown in figures (2 and 3) confirm the fact that dilute polymer solutions exhibit large reductions in both heat and momentum transfer in turbulent flows. Many physical models have been proposed to understand the phensmenon Virk [2] . Non is capable of explaining its different aspects. In a trial to do so, we have to understand the mechanism by which momentum and heat are transfered from the wall to the core of the flow and possible influence of drag reducing additives upon such mechanism.

Recent studies [21 - 23] show that the flow in the near wall region is characterized by a structure of countre - rotating vortices which has a quasi - cyclic rature. It grows in a similar way to that of the laminar boundary layer over a flat plate, until it reaches certain thickness. Then, it starts to oscillate and finally detaches from the wall and ejects out to the core of the flow. Such phase is followed by a sweep action of the fluid incoming from the core to the wall region to replace the ejected one and start a new cycle. The momentum and energy transfered by molecular diffusivity during the development phase are transfered to the flow during the ejection - sweep phase of the cycle. The frequency of occurence of such process is actually a measure of the rate of transfer in turbulent flows [22] . It has been, experimentally, found that the presence of macromolecules increases the resistance of the flow to elongational deformation and vortex stretching. Therefore, it is thought that polymer additives decrease the rate of development of the flow structure nearby the surface. Hence, a substantial increase in the time between burst ejections is experimentally found [3] in drag reducing additive flows compared with Newtonian ones. This results in the large decrease in the rate of transfer of momentum, heat and mass. Due to the decrease in the frequency of the viscous layer ejections, an increase in the thickness of the viscous layer is experimentally found and an increase in the van Driest damping coefficient A^+ is assumed as given in relation (18). In fact, experimental results confiom the analogy between heat and momentum transfer assumed in the mathematical model and explained by the physical model discussed above.

6. CONCLUSIONS

This work presents on experimental and theoretical analysis for constant wall temperature heat transfer in dilute polyacrylamide solutions. The results exhibt substantial reductions in both friction and heat transfer compared with water (Newtonian) flow. It confirms the analogy between momentum and heat transfer in drag reducing additive fluid flows. This actually tends to substantiate the thought that drag reducing additives damp the turbulent motion near the wall reducing the frequency of flow ejections to the core. As a result, a reduction in the rate of transfer of momentum and heat occurs.

REFERENCES

1 Hoyt, J.W., "The Effect of Additives on Fluid Friction", Trans. ASME, 94D, 258, (1972).

2 Virk, P.S., "Drag Reduction Fundamentals" AIchE J., 21, 625(1975).

3 Mabie, L.M., "Drag Reduction in Turbulent Shear Flow Due to Injected Polymer Solutions", Ph.D. Thesis, Edinburgh University U.K. (1978).

4 Gupta M.K., Metzner, A.B., and Hartnett, J.P., "Turbulent Heat Transfer Characteristics of Viscoelastic Fluids" Int. J. of Heat and Mass Transfer, 10, 1211, (1967).

5 Metzner, A.B., and Friend P.S., "Heat Transfer to Turbulent non Newtonian Fluids". Ind. Eng. Chem. 51, 879 (1959).

6 Debrule, P.M., and Sabersky, R.H., "Heat Transfer and Friction Coefficients in smooth and Rough Tubes with Dilute Polymer Solutions", Int J. Heat Mass Transfer, 17, 529, (1972).

7 Smith, K.A., Keuroghlian, G.H., Virk, P.S., and Merrill E.W., "Heat Transfer to Drag Reducing Polymer Solutions", AIChE J., 15, 294, (1969).

8 Mc Nally, W.A., "Heat and Momentum Transport in Dilute Polyethylene Oxide Solutions" Ph.D. Thesis, University of Rhode Island (1968).

9 Kwack E.Y., Cho, Y.I., and Hartnett J.P., "Heat Transfer to Polyacrylamide Solutions in Turbulent Pipe Flow" AIChE J., 27, 123 (1981).

10 Kader, B.A., and Yaglom, A.M., "Heat and Mass Transfer Laws for Fully Turbulent Wall Flows" Int. J. Heat Mass Transfer, 15, 2329, (1972).

11 Chajar, A.J., and Tiederman, W.G., "Prediction of Heat Transfer Coefficients in Drag Reducing Turbulent Pipe Flows." AIChE J., 23, 128, (1977).

12 Dimant, Y. and Poreh, "Heat Transfer in flows with Drag Reduction", Advances in Heat Transfer, Vol. 12, Academic Press, New York (1976).

13 Kale, D.D., "An Analysis of Heat Transfer to Turbulent Flow of Drag Reducing Fluids", Int. J. Heat Mass Transfer, 20, 1077 (1977).

14 Smith, R., and Edwards, M.F., "Heat Transfer to non Newtonian and Drag Reducing Fluids in Turbulent Pipe flow" Int. J. Heat Mass Transfer, 24, 1069, (1981).

15 Yoon, H.K., and Chajar, A.J., "Analytical Study of Turbulent Heat Transfer to Polymer Solutions in Tubes with Constant Heat Flux" Proceedings of the ESRC conference, New Orleans, Louisiana, Feb. 12-16, (1984). Laminar Turbulent Boundary Layers Symposium.

16 Cess, R.D., "A Survey of the Literature in Heat Transfer in Turbulent Tube Flow", Westinghouse Res. Rept. 8-0529-R24, Westinghouse Corporation, Philadelphia, Pa. (1958).

17 Van Driest, E.R., "On Turbulent Flow Near a Wall", J. Aero. Sci. 23, 1007, (1956).

18 Mizushina, T., and Ogino, F., "Eddy Viscosity and Universal Velocity Profile in Turbulent Flow in a Straight Pipe", J. Chem. Engng. Japan, 3, 166 (1970).

19 Mizushina, T., and Usui, H. "Reduction of Eddy Diffusion for Momentum and Heat in Viscoelastic Fluid Flow in Circular Tube". The Physics of Fluids, 20, 5100, (1977).

20 Rabie, L.H., "An Analytical Model for Turbulent Heat and Mass Transfer in Non Newtonian Fluid Flows", To be Published.

21 Corino, E.R., and Brodkay, R.S., J. Fluid Mech., 37, 1, (1969).

22 Kim, H.T., Kline, S.J., and Reynolds, W.C., J. Fluid Mechanics. 50, 133, (1971).

23 Rabie, L.H., "The periodic viscous sublayer in Turbulent Flow of Drag Reducing Additives", Proceedings of the 4th Int. Conference for Power Mechanical Engineering, Menof University, Egypt (1982).